Orthogonal Generalized Higher Reveres Derivations on Semi prime Rings

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Abstract

In this paper we introduce and study the concept of orthogonal generalized higher reveres derivations on semi prime rings , also we prove the following theorem if D_n and G_n are orthogonal, then the following relations hold:

(1)$D_n(x)G_n(y) = G_n(x)D_n(y) = 0$
(2)d_n and G_n are orthogonal higher reveres derivations and $d_n(x)G_n(y) = G_n(y)d_n(x) = 0$, for all $x, y, \in R$.
(3)g_n and D_n are orthogonal higher reveres derivations and$g_n(x)D_n(y) = D_n(y)g_n(x) = 0$, for all $x, y, \in R$.
(4)In addition to demonstration other.

Keywords

Semiprimering, Generalized higher reveres derivations, Orthogonal generalized higher reveres derivations.

1. Introduction

Throughout this paper R will represent an associative ring. R is said to be 2-torsion free if 2n=0 , $n \in R$ implies $n = 0$, R is prime if $aRb = (0)$implies $a = 0$ or $b = 0$, and R is simeprime if $aRa = (0)$implies $a = 0$.
In [5] I.N.Herstein defined derivation (resp. Jordan derivation) of R is an additive mapping $d: R \to R$ such that $d(ab) = d(a)b + ad(b)$, for every $a, b \in R$(resp.$d(a^2) = d(a)a + ad(a)$ for every $a \in R$).As it is well known that every derivation is Jordan derivation and the converse is in general not true.
B. Havala in [4] defined a generalized derivation and Jordan generalized derivation as follow for all $x, y \in R$ then the additive mapping $f: R \to R$ is called a generalized derivation if there exists a derivation $d: R \to R$ such that $f(xy) = f(x)y + yd(x)$, we called an additive mapping$f: R \to R$ a Jordan generalized derivation if there exists a Jordan derivation$d: R \to R$ such that $f(x^2) = f(x)x + xd(x)$ for all $x \in R$.
M. Brešar and J. Vukman in [2] defined reverse derivation as following for every $x, y \in R$ then the additive mapping $d: R \to R$ is called reverse derivation on R, $d(xy) = d(y)x + yd(x)$, d is called Jordanreverse derivation on R if for every $x \in R$, $d(x^2) = d(x)x + xd(x)$.

M.R. Salih in [7] defined generalized reverse derivation as follows for all $x, y \in R$ then additive mapping of a ring R into itself then f is called generalized reverse derivation on R if there exists a reverse derivation d on R such that $f(xy) = f(y)x + yd(x)$.
M.R. Salih in [7] introduced the concept of higher reverse derivations on rings as follows let $D = (di)_{i \in N}$ be a family of additive mappings on R, such that $d_0 = id_R$ then D is called a higher reverse derivation on R if for every $x, y \in R$ and $n \in$ N , $d_n(xy) = \sum_{i+j=n} d_i(y)d_j(x)$ .And defined generalized higher reveres derivation on rings as follows let $F = (f_i)_{i \in N}$ be a family of additive mappings of a ring R into itself such that $f_0 = id_R$ then F is called a generalized higher reveres derivations on$R$if there exists a higher reveres derivation$D = (d_i)_{i \in N}$ on R such that for all $n \in$ N we have$f_n(xy) = \sum_{i+j=n} f_i(y)d_j(x)$.

Brešar and Vukman in [3] defined orthogonal derivations in semiprime rings as follows two derivations D and G on a ring R are said to be orthogonal if $D(x)RG(y) = (0) = G(y)RD(x)$, for all $x, y \in R$.

N. Argac and A. Nakajima and E. Albas in [1] extend the results of derivation into generalized derivation of R by presenting the concept of orthogonal generalized derivation.

A.H. Majeed in[6] defined orthogonal reverse derivation on ring as follows two reverse derivations f and h of R are called orthogonal if $f(x)Rh(y) = (0) = h(y)Rf(x)$ for all $x, y \in R$.In this paper, the concept of orthogonal generalized higher reveres derivations is present, some characterization of it on semiprime rings are obtained we also investigate conditions for two generalized higher reveres derivations to be orthogonal.

2. Orthogonal generalized higher reverse derivations on semi prime rings

In this section we introduce and study the concept of orthogonal generalized higher reverse derivations on rings.

Definition (2.1)

Let R be a ring, two generalized higher reverse derivations
$D = (D_i)_{i\in N}$ and $G = (G_i)_{i\in N}$ on R are called orthogonal if for every $x, y \in R$ and $n \in N$,then

$$D_n(x)RG_n(y) = (0) = G_n(y)RD_n(x)$$

Where

$$D_n(x)RG_n(y) = \sum_{i=1}^{n} D_i(x)RG_i(y) = (0)$$

The following is an example of orthogonal generalized higher revers derivations on semiprime rings.

Example (2.2)

Let R be a ring and $d = (d_i)_{i\in N}$, $g = (g_i)_{i\in N}$ be two higher reverse derivations of R. We put $R' = \{(a, b): a, b \in R$ then the mapping $d'=d_i' i\in N$ and $g'=g_i' i\in N$ from R' into itself which are defined by
$d_n'((x, y)) = (d_n(x), 0)$and
$g_n'((x, y)) = (0, g_n(y))$, for $(x, y) \in R'$ are higher reverse derivations of R'
Moreover if (D_n, d_n) and (G_n, g_n) are generalized higher reverse derivations on R and we defined the mappings $D' = (D_i')_{i\in N}, G' = (G_i')_{i\in N}$ on R' by $D_n'((x, y)) = (D_n(x), 0)$ and $G'((x, y)) = (0, G_n(y))$, for all $(x, y) \in R'$
Then, (D_n', d_n') and (G_n', g_n') are generalized higher reverse derivations of R' and D_n', G_n' are orthogonal.

Lemma (2.3):[3]

Let R be a 2-torsion free semiprime ring and a,b are elements of R then the following conditions are equivalent for all $x \in R$,
1.$axb = 0$
2.$bxa = 0$
3.$axb + bxa = 0$

If one of these conditions is fulfilled then $ab = ba = 0$.

Theorem (2.4)

Let $D = (D_i)_{i\in N}$ and $G = (G_i)_{i\in N}$ be generalized higher reverse derivations on a 2-torison free semiprime ring, where $D_n and G_n$ are commuting mappings if D_n and G_n are orthogonal. Then the following relations hold:
(i) $D_n(x)G_n(y) = G_n(x)D_n(y) = 0$
Hence $D_n(x)G_n(y) + G_n(x)D_n(y) = 0$, for all $x, y \in R$.
Proof: By the hypothesis, we have

$$\sum_{i=1}^{n} D_i(x)zG_i(y) = 0, \text{ for all} x, y, z \in R \text{by Lemma (2.3) we get}$$

$$D_n(x)G_n(y) = G_n(x)D_n(y) = 0$$

Hence $D_n(x)G_n(y) + G_n(x)D_n(y) = 0$
(ii) d_n and G_n are orthogonal higher reverse derivations and

$$d_n(x)G_n(y) = G_n(y)d_n(x) = 0 \text{ for all} x, y \in R$$

Proof:

Since $D_n(x)G_n(y) = 0 = G_n(x)D_n(y)$ by (i)

$$\sum_{i=1}^{n} D_i(x)G_i(y) = 0$$

Replace x by xy we get

$$\sum_{i=1}^{n} D_i(xy)G_i(y) = 0$$

$$\sum_{i=1}^{n} D_i(y)d_i(x)G_i(y) = 0$$
$$\sum_{i=1}^{n} G_i(y)d_i(x)D_i(y) = 0$$

Replace $D_i(y)$ by $G_i(y)$ and $d_i(x)$ by$d_i(x)z$

$\sum_{i=1}^{n} G_i(y)d_i(x)zG_i(y) = 0$ And multiply of right by $d_i(x)$ we get

$$\sum_{i=1}^{n} G_i(y)d_i(x)zG_i(y)\, d_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} G_i(y)d_i(x) = 0$$

$$\sum_{i=1}^{n} d_i(x)G_i(y) = 0 \Longrightarrow d_n(x)G_n(y) = 0 \qquad \dots (1)$$

Also replace x and y by xy in $G_n(y)D_n(x) = 0$

$$\sum_{i=1}^{n} G_i(xy)D_i(xy) = 0$$
$$\sum_{i=1}^{n} G_i(y).\, g_i(x).\, D_i(y).\, d_i(x) = 0$$

Replace $g_i(x)$ by $d_i(x)z$ and $D_i(y)$ by $G_i(y)$

$$\sum_{i=1}^{n} G_i(y)d_i(x)zG_i(y)d_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} G_i(y)d_i(x) = 0$$

$$G_n(y)d_n(x) = 0 \qquad \dots (2)$$

Form (1) and (2) we get

$$d_n(x)G_n(y) = G_n(y)d_n(x) = 0$$

(iii) g_n and D_n are orthogonal higher reverse derivations and

$$g_n(x)D_n(y) = D_n(y)g_n(x) = 0, \text{ for all } x, y \in R$$

Proof: by (i) we have

Since $D_n(x)G_n(y) = 0 = G_n(x)D_n(y)$

$$\sum_{i=1}^{n} G_i(x)D_i(y) = 0$$

Replace x by xy we get

$$\sum_{i=1}^{n} G_i(xy)D_i(y) = 0$$
$$\sum_{i=1}^{n} G_i(y)g_i(x)D_i(y) = 0$$
$$\sum_{i=1}^{n} D_i(y)g_i(x)G_i(y) = 0$$

Replace $g_i(x)$ by$g_i(x)z$ and$G_i(y)$ by $D_i(y)$

$$\sum_{i=1}^{n} D_i(y)g_i(x)zD_i(y) = 0$$

And multiply of right by $g_i(x)$ we get

$$\sum_{i=1}^{n} D_i(y)g_i(x)zD_i(y)\, g_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} D_i(y)g_i(x) = 0 \Longrightarrow \sum_{i=1}^{n} g_i(x)D_i(y) = 0$$

$$g_n(x)D_n(y) = 0 \qquad \ldots (1)$$

Also $D_n(x)G_n(y) = 0$

Replace x by xy and y by xy we get

$$\sum_{i=1}^{n} D_i(xy)G_i(xy) = 0$$

$$\sum_{i=1}^{n} D_i(y).\, d_i(x).\, G_i(y).\, g_i(x) = 0$$

Replace $d_i(x)$ by $g_i(x)z$ and $G_i(y)$ by $D_i(y)$ we get

$$\sum_{i=1}^{n} D_i(y)g_i(x)zD_i(y)g_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} D_i(y)g_i(x) = 0$$

$$D_n(y)g_n(x) = 0 \qquad \ldots (2)$$

Form (1) and (2) we get

$$g_n(x)D_n(y) = D_n(y)g_n(x) = 0$$

(iv) d_n and g_n are orthogonal higher reverse derivations.

Proof: By (i) we have

Since $D_n(x)G_n(y) = 0 = G_n(x)D_n(y)$ by (i)

$$\sum_{i=1}^{n} D_i(x)G_i(y) = 0$$

Replace x by xy and y by yx we get

$$\sum_{i=1}^{n} D_i(xy)G_i(yx) = 0$$

$$\sum_{i=1}^{n} D_i(y)d_i(x)G_i(x)g_i(y) = 0$$

Replace $D_i(y) by g_i(y) z and G_i(x) by G_i(x)z$

$$\sum_{i=1}^{n} g_i(y)zd_i(x).\, G_i(x)zg_i(y) = 0$$

Replace $G_i(x) by zd_i(x)$ we get

$$\sum_{i=1}^{n} g_i(y)zd_i(x)zd_i(x)zg_i(y) = 0$$

$$\sum_{i=1}^{n} d_i(x)zg_i(y)zd_i(x)zg_i(y) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} d_i(x)zg_i(y) = 0$$

Then d_n and g_n are orthogonal.

(v) $d_n G_n = G_n d_n = 0$ and $g_n D_n = D_n g_n = 0$, where D_n, G_n are commuting mappings.

Proof: By (ii) we have

Since $d_n(x)G_n(y) = 0$,

Therefore $\sum_{i=1}^{n} d_i(x).G_i(y) = 0$

Replace y by yx we get

$$\sum_{i=1}^{n} d_i(x).G_i(yx) = 0$$

$$\sum_{i=1}^{n} d_i(x).G_i(x)g_i(y) = 0$$

$$\sum_{i=1}^{n} G_i(x).d_i(x).g_i(y) = 0$$

Replace $d_i(x)$ by $d_i(x)z$ and $g_i(y)$ by $G_i(x)$

$\sum_{i=1}^{n} G_i(x).d_i(x)zG_i(y) = 0$ And multiply of right by $d_i(x)$ we get

$$\sum_{i=1}^{n} G_i(x).d_i(x)zG_i(x)d_i(x) = 0$$

Since R is semiprime ring

Therefore $\sum_{i=1}^{n} G_i(x)d_i(x) = 0 \Longrightarrow G_n d_n = 0$

Also since $G_n(x)d_n(y) = 0$, by(ii)

$$\sum_{i=1}^{n} G_i(x)d_i(y) = 0$$

Replace x by xy we get

$$\sum_{i=1}^{n} G_i(xy)d_i(y) = 0$$

$$\sum_{i=1}^{n} G_i(y).g_i(x).d_i(y) = 0$$

$$\sum_{i=1}^{n} d_i(y).G_i(y).g_i(x) = 0$$

replace $g_i(x)$ by $zd_i(y)$ we get

$\sum_{i=1}^{n} d_i(y).G_i(y).zd_i(y) = 0$ and multiply of the right by $G_i(y)$ we get

$$\sum_{i=1}^{n} d_i(y).G_i(y).zd_i(y)G_i(y) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} d_i(y)G_i(y) = 0 \Longrightarrow d_n G_n = 0$$

And since $g_n(x)D_n(y) = 0$, by (iii)

$$\sum_{i=1}^{n} g_i(x)D_i(y) = 0,$$

Replace$y$byyxwe get

$$\sum_{i=1}^{n} g_i(x)D_i(yx) = 0$$
$$\sum_{i=1}^{n} g_i(x)D_i(x)d_i(y) = 0$$
$$\sum_{i=1}^{n} D_i(x)g_i(x)d_i(y) = 0$$

Replace $d_i(y)$ by $zD_i(x)$ we get
$\sum_{i=1}^{n} D_i(x)g_i(x)zD_i(y) = 0$, and multiply of right by $g_i(x)$ we get

$$\sum_{i=1}^{n} D_i(x)g_i(x)zD_i(x)\, g_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} D_i(x)g_i(x) = 0 \Longrightarrow D_n g_n = 0$$

And since $D_n(x)g_n(y) = 0$

$$\sum_{i=1}^{n} D_i(x)g_i(y) = 0,$$

Replace$x$byxywe get

$$\sum_{i=1}^{n} D_i(xy)g_i(y) = 0$$
$$\sum_{i=1}^{n} D_i(y)d_i(x)g_i(y) = 0$$
$$\sum_{i=1}^{n} g_i(y)D_i(y)d_i(x) = 0$$

Replace $d_i(x)$ by $zg_i(x)$ we get
$\sum_{i=1}^{n} g_i(y)D_i(y)zg_i(x) = 0$, and multiply of right by $D_i(y)$we get

$$\sum_{i=1}^{n} g_i(y)D_i(y)zg_i(y)D_i(y) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} g_i(y)D_i(y) = 0 \Longrightarrow g_n D_n = 0$$

(vi) $D_n G_n = G_n D_n = 0$ where $D_n and G_n$ are commuting mappings
Proof:
Since $D_n(x)G_n(y) = 0$

$$\sum_{i=1}^{n} D_i(x)G_i(y) = 0$$

Replace y by yx we get

$$\sum_{i=1}^{n} D_i(x)G_i(yx) = 0$$
$$\sum_{i=1}^{n} D_i(x)G_i(x)g_i(y) = 0$$

$$\sum_{i=1}^{n} G_i(x)D_i(x)g_i(y) = 0$$

Replace $g_i(y)$ by $zG_i(x)$ we get

$$\sum_{i=1}^{n} G_i(x)D_i(x)zG_i(x) = 0$$

And multiply of right by $D_i(x)$ we get

$$\sum_{i=1}^{n} G_i(x)D_i(x)zG_i(y)D_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} G_i(x)D_i(x) = 0 \Longrightarrow G_n D_n = 0$$

Since therefore $G_n(x)D_n(y) = 0$Since D_n and G_n are commuting
Therefore $D_n G_n = 0$

3. Main results:

In this section we present the main results of this paper we begin with the following lemma.

Lemma (3.1): Let R be a semiprime ring, U be an ideal of R and $V = Ann(U)$. If $D = (D_i)_{i\in N}$ is higher reverse derivations associated with a generalized higher reverse derivation of R such that $d = (d_i)_{i\in N}$ and d_n is commuting mapping, $D_n(R), d_n(R) \subset U$ then $D_n(V) = d_n(V) = 0$.

Proof: If $x \in V$, then $xU = (0)$ by the hypothesis we have:

$$d_n(R) \subset U \Longrightarrow d_n(U) \subset U$$

Hence $D_n(xy) = 0$

$$\sum_{i=1}^{n} D_i(y)d_i(x) = 0$$

Replace y by yx we get

$$\sum_{i=1}^{n} D_i(yx)d_i(x) = 0$$

$$\sum_{i=1}^{n} D_i(x)d_i(y)d_i(x) = 0$$

$$\sum_{i=1}^{n} D_i(x)d_i(x)d_i(y) = 0$$

Replace $d_i(y)$ by $D_i(x)$ we get

$$\sum_{i=1}^{n} D_i(x)d_i(x)D_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} D_i(x) = 0 \Longrightarrow D_n(x) = 0$$

Since $D_n(x) \in U \cap V$, we get $D_n(V) = 0$
Similarly
If $x \in V$ then $xU = (0)$

$$D_n(R) \subset U \Longrightarrow D_n(U) \subset U$$

$$\text{Hence}d_n(xy) = 0 \Longrightarrow \sum_{i=1}^{n} d_i(y).\, d_i(x) = 0$$

Replace y by yx we get

$$\sum_{i=1}^{n} d_i(yx).\, d_i(x) = 0$$

$$\sum_{i=1}^{n} d_i(x).\, d_i(y).\, d_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} d_i(x) = 0$$

$d_n(x) \in U \cap V$, we get

$$d_n(x) = 0 \Longrightarrow d_n(V) = 0$$

Lemma (3.2): Let (D_n, d_n) be generalized higher reverse derivations of semiprime ring R. If $D_n(x)D_n(y) = 0$, for all $x, y \in R$ and $n \in N$then $D_n = d_n = 0$.

Proof: By the hypothesis, we have

$$D_n(x)D_n(y) = 0 \Longrightarrow \sum_{i=1}^{n} D_i(x)D_i(y) = 0$$

replace y by xywe get

$$\sum_{i=1}^{n} D_i(x)D_i(xy) = 0$$

$$\sum_{i=1}^{n} D_i(x)D_i(y)d_i(x) = 0$$

Replace $D_i(x)$ by $d_i(x)$

$$\sum_{i=1}^{n} d_i(x)D_i(y)d_i(x) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} d_i(x) = 0 \Longrightarrow d_n = 0$$

Also by the hypothesis we have

$$\sum_{i=1}^{n} D_i(x)D_i(y) = 0$$

replace x by xywe get

$$\sum_{i=1}^{n} D_i(xy)D_i(y) = 0$$

$$\sum_{i=1}^{n} D_i(y)d_i(x)D_i(y) = 0$$

Since R is semiprime ring

$$\sum_{i=1}^{n} D_i(y) = 0 \Longrightarrow D_n = 0$$

We get $D_n = d_n = 0$

Theorem (3.3): Let R be a 2-torsion free semiprime ring, let $D = (D_i)_{i\in N}$ and $G = (G_i)_{i\in N}$ generalized higher reverse derivations associated with higher reverse derivations $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$ respectively, where D_n and G_n are orthogonal iff for all $x, y \in R$ then:

(a) $D_n(x)G_n(y) + G_n(x)D_n(y) = (0)$

(b) $d_n(x)G_n(y) + g_n(x)D_n(y) = (0)$

Proof: Suppose that $D_n andG_n$ are orthogonal

To prove (a) $D_n(x)G_n(y) + G_n(x)D_n(y) = (0)$
$\sum_{i=1}^{n} D_i(x)zG_i(y) = \sum_{i=1}^{n} G_i(x)zD_i(y) = (0)$ where $z \in R$

$$\sum_{i=1}^{n} D_i(x)zG_i(y) = \sum_{i=1}^{n} G_i(x)zD_i(y) = (0)$$

Hence By lemma (2.3)
$D_n(x)G_n(y) + G_n(x)D_n(y) = 0$
Now, to prove (b) $d_n(x)G_n(y) + g_n(x)D_n(y) = (0)$
Since by (a) $D_n(x)G_n(y) = (0)$

$$\sum_{i=1}^{n} d_i\big(D_i(x)G_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big)d_i\big(D_i(x)\big) = (0)$$

Replace $D_i(x)$ by $G_i(x)$ we get

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big)d_i\big(G_i(x)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big)G_i\big(d_i(x)\big) = (0)$$

Replace $G_i(y)$ by x and $d_i(x)$ by y we get

$$\sum_{i=1}^{n} d_i(x)G_i(y) = (0)$$

$Hence$
$d_n(x)G_n(y) = (0)$ … (1)
Since by (a) $G_n(x)D_n(y) = (0)$

$$\sum_{i=1}^{n} g_i\big(G_i(x)D_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} g_i\big(D_i(y)\big)g_i\big(G_i(x)\big) = (0)$$

Replace $G_i(x)$ by $D_i(y)$ we get

$$\sum_{i=1}^{n} g_i\big(D_i(y)\big)g_i\big(D_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} g_i\big(D_i(y)\big)D_i\big(g_i(y)\big) = (0)$$

Replace $D_i(y)$ by x and $g_i(y)$ by y we get

$$\sum_{i=1}^{n} g_i(x)D_i(y) = (0)$$

Hence
$g_n(x)D_n(y) = (0)$ … (2)
From (1) and (2) we get

$$d_n(x)G_n(y) + g_n(x)D_n(y) = (0)$$

Conversely: Suppose that (a) we get

$$D_n(x)G_n(y) + G_n(x)D_n(y) = (0)$$

Replace x by yx we get

$$\sum_{i=1}^{n} D_i(yx)G_i(y) + G_i(yx)D_i(y) = (0)$$

$$\sum_{i=1}^{n} D_i(x).\, d_i(y).\, G_i(y) + G_i(x).\, g_i(y).\, D_i(y) = (0)$$

Replace $g_i(y)$ by $d_i(y)$ we get

$$\sum_{i=1}^{n} D_i(x).\, d_i(y).\, G_i(y) + G_i(x).\, d_i(y).\, D_i(y) = (0)$$

$$\sum_{i=1}^{n} D_i(x).\, d_i(y).\, G_i(y) = 0$$

$$\sum_{i=1}^{n} G_i(y).\, d_i(y).\, D_i(x) = 0$$

Then D_n, G_n are orthogonal

<u>Theorem (3.4)</u>Let R be a 2-torsion free semiprime ring. let $D = (D_i)_{i\in N}$ and $G = (G_i)_{i\in N}$ are generalized higher reverse derivations associated with $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$ are higher reverse derivations, of D_n *and* G_n respectively where D_n, G_n are commuting mappings, then D_n and G_n are orthogonal iff $D_n(x)G_n(y) = d_n(x)G_n(y) = (0)$ for all $x, y \in R$.

Proof: Suppose that D_n, G_n are orthogonal by Theorem (2.4) part (i) we get $D_n(x)G_n(y) = 0$

$$\sum_{i=1}^{n} D_i(x)G_i(y) = 0$$

$$\sum_{i=1}^{n} d_i\big(D_i(x)G_i(y)\big) = 0$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big).\, d_i\big(D_i(x)\big) = 0$$

Replace $D_i(x)$ by $G_i(x)$ we get

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big).\, d_i\big(G_i(x)\big) = 0$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big).\, G_i\big(d_i(x)\big) = 0$$

Replace $G_i(y)$ by x and $d_i(x)$ by y

$$\sum_{i=1}^{n} d_i(x).\, G_i(y) = 0$$

$$d_n(x)G_n(y) = 0$$

Conversely

Since $D_n(x)G_n(y) = (0)$

$$\sum_{i=1}^{n} D_i(x).\, G_i(y) = (0)$$

Replace x by yx we get

$$\sum_{i=1}^{n} D_i(yx).\, G_i(y) = (0)$$

$$\sum_{i=1}^{n} D_i(x).\, d_i(y).\, G_i(y) = (0)$$

$$\sum_{i=1}^{n} G_i(y).\,d_i(y).\,D_i(x) = (0)$$

Hence $\sum_{i=1}^{n} D_i(x).\,d_i(y).\,G_i(y) = (0) = \sum_{i=1}^{n} G_i(y).\,d_i(y).\,D_i(x)$

Then D_n and G_n are orthogonal

Theorem (3.5) Let R be a 2-torsion free semiprime ring, $D = (D_i)_{i\in N}$ and $G = (G_i)_{i\in N}$ are generalized higher reverse derivations associated with higher reverse derivations $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$, where d_n, G_n are commuting mappings, then D_n and G_n are orthogonal iff $D_n(x)G_n(y) = (0)$ for all $x, y \in R$ and $d_n G_n = d_n g_n = 0$.

Proof: Suppose that D_n and G_n are orthogonal.

$D_n(x)G_n(y) = 0$ we get by Theorem (2.4) part (i) and part (ii) $G_n(y)d_n(x) = 0$

$$\sum_{i=1}^{n} d_i\big(G_i(y).\,d_i(x)\big) = 0$$

$$\sum_{i=1}^{n} d_i\big(d_i(x)\big).\,d_i\big(G_i(y)\big) = 0$$

Replace $d_i(x)$ by $zG_i(y)$ we get

$$\sum_{i=1}^{n} d_i\big(zG_i(y)\big).\,d_i\big(G_i(y)\big) = 0$$

$\sum_{i=1}^{n} d_i\big(G_i(y)\big).\,d_i(z).\,d_i\big(G_i(y)\big) =0$

Since R is semiprime

$$\sum_{i=1}^{n} d_i\big(G_i(y)\big) = (0)$$

$$d_n G_n = 0 \qquad \ldots(1)$$

By Theorem (2.4) part (ii) $d_n(x)G_n(y) = (0)$

$$\sum_{i=1}^{n} g_i\big(d_i(x).\,G_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} g_i\big(G_i(y)\big).\,g_i\big(d_i(x)\big) = (0)$$

Replace $G_i(y)$ by $zd_i(x)$ we get

$$\sum_{i=1}^{n} g_i(zd_i(x)).\,g_i\big(d_i(x)\big) = (0)$$

$$\sum_{i=1}^{n} g_i\big(d_i(x)\big).\,g_i(z).\,g_i\big(d_i(x)\big) = (0)$$

Since R is semiprime we get

$$\sum_{i=1}^{n} g_i\big(d_i(x)\big) = 0$$

$$\sum_{i=1}^{n} d_i\big(g_i(x)\big) = (0)$$

$$d_n g_n = 0 \qquad \ldots(2)$$

From (1) and (2) we get

$$d_n G_n = d_n g_n = 0$$

Conversely: Suppose that $d_n G_n = 0$

$$\sum_{i=1}^{n} d_i\big(G_i(xy)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(G_i(y)g_i(x)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(g_i(x)\big).\,d_i\big(G_i(y)\big) = (0)$$

$$\sum_{i=1}^{n} d_i\big(g_i(x)\big).\,G_i\big(d_i(y)\big) = (0)$$

Replace $g_i(x)$ by x and $d_i(y)$ by y

$$\sum_{i=1}^{n} d_i(x).\,G_i(y) = (0)$$

$$d_n(x)G_n(y) = 0$$

By Theorem (2.8) we get D_n and G_n are orthogonal.

Theorem (3.6) Let R be a 2-torsion free semiprime ring and D_n, G_n are generalized higher reverse derivations of R. Suppose $D_n^2 = G_n^2$ then $D_n - G_n$ and $D_n + G_n$ are orthogonal.

Proof: Since $D_n^2 = G_n^2$ we get

$$\sum_{i=1}^{n}\big((D_i + G_i)(D_i - G_i) + (D_i - G_i)(D_i + G_i)\big)(x)$$

$$= \sum_{i=1}^{n}((D_i + G_i)(D_i - G_i))(x) + ((D_i - G_i)(D_i + G_i))(x)$$

$$= \sum_{i=1}^{n}(D_i + G_i)(x)(D_i - G_i)(x) + \sum_{i=1}^{n}(D_i - G_i)(x)(D_i + G_i)(x)$$

$$= \sum_{i=1}^{n}\big(D_i(x) + G_i(x)\big)(D_i(x) - G_i(x)) + \sum_{i=1}^{n}\big(D_i(x) - G_i(x)\big)(D_i(x) - G_i(x))$$

$$= \sum_{i=1}^{n} D_i^2(x) - (D_iG_i)(x) + (G_iD_i)(x) - G_i^2(x) + \sum_{i=1}^{n} D_i^2(x) + (D_iG_i)(x) - (G_iD_i)(x) - G_i^2(x) = 0$$

$$= \sum_{i=1}^{n} D_i^2(x) - (D_iG_i)(x) - G_i^2(x) + D_i^2(x) - (D_iG_i)(x) + G_i^2(x) = 0$$

Therefore $\sum_{i=1}^{n}(D_i + G_i)(x)(D_i - G_i)(x) + (D_i - G_i)(x)(D_i + G_i)(x) = 0$

By Lemma (2.3) we get

$$\sum_{i=1}^{n}(D_i + G_i)(x)(D_i - G_i)(x) = 0 = \sum_{i=1}^{n}(D_i - G_i)(x)(D_i + G_i)(x)$$

Hence $D_n - G_n$ and $D_n + G_n$ are orthogonal.

Corollary (3.7): Let R be a 2-torsion free semiprime ring and D_n, G_n be generalized higher derivations on R. Suppose $D_n^2 = D_n^2$ then $D_n - G_n$ and $D_n + G_n$ are orthogonal.

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