On The Evolution of Random Hyper Tangle Graph

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Abstract

In this paper, we will study random hyper tangle graph .we will evolution the probability of hyper tangle graph. We introduce the number of possible edges in hyper tangle graph.

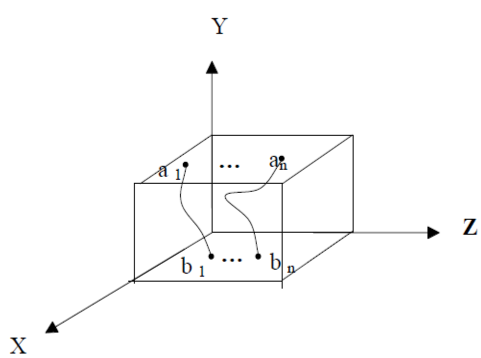
Keywords: Hyper graph, tangle graph, hyper tangle graph, probability, and distribution function.

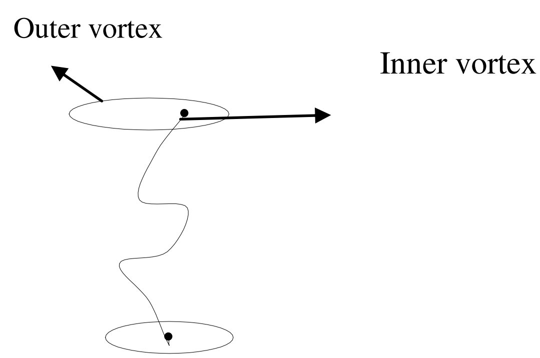
1. *Introduction*

In mathematics, **random** graph is the general term to refer to probability distributions over graphs. Random graphs may be described simply by a probability distribution, or by a random process which generates them [1]. The theory of random graphs lies at the intersection between graph theory and probability theory. From a mathematical perspective, random graphs are used to answer questions about the properties of typical graphs In a mathematical context, random graph refers almost exclusively to the Erdős–Rényi random graph model. In other contexts, any graph model may be referred to as a random graph. Random graphs were first defined by Paul Erdős and Alfréd Rényi in their 1959 paper "On Random Graphs [2]. And independently by Gilbert in his paper "Random graphs"[3].

Definition 1:

Tangle graph: Let D be a unit cube, so D={(x,y,z):0<x,y,z<1} on the top face of cube place n points $a_1,a_2,\ldots,a_n$ similarly place on bottom face $b_1,b_2,\ldots,b_n$, now join the points $a_1,a_2,\ldots,a_n$ with $b_1,b_2,\ldots,b_n$ by arcs $d_1,d_2,\ldots,d_n$ these arcs are disjoint and each d_i connects some a_j to b_k not connect a_j to a_k or b_j to b_k this called tangle [1].





Definition 2:

Hyper graph: A hyper graph is a graph which an edge can connect any number of vertices. Formally, a hyper graph ***H*** is a pair ***H*** =(X,E) Where X is a set of elements called nodes or vertices, and E is a set of non-empty sub set of X called hyper edge or edges[2].

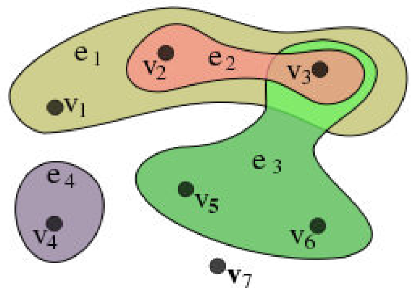
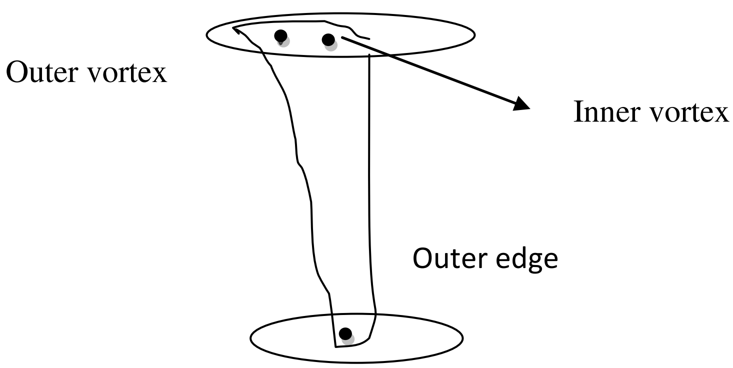


Fig.1 An example of a hyper graph, with

$X = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$And $E = \{e_1, e_2, e_3, e_4\}$ =$\{\{v_1, v_2, v_3\}, \{v_2, v_3\}, \{v_3, v_5, v_6\}, \{v_4\}\}$.

Tangle hyper graph:

A graph T_h=(V,E) whose vertices consists of inner and outer vertices the outer vertices looks like the hyper edge of hyper graph and set of inner and outer edges.



Erdős and Rényi model:

- In the $G(n, M)$ model, a graph is chosen uniformly at random from the collection of all graphs which have n nodes and M edges. For example, in the $G(3, 2)$ model, each of the three possible graphs on three vertices and two edges are included with probability 1/3 [3].

Edgar Gilbert model:

- In the $G(n, p)$ model, a graph is constructed by connecting nodes randomly. Each edge is included in the graph with probability p independent from every other edge. Equivalently, all graphs with n nodes and M edges have equal probability of

$$p^M(1-p)^{\binom{n}{2}-M}.$$ **[3]**

Average number of edge:

The expected number of edges in $G(n, p)$ is $\binom{n_{in}}{k} + \binom{\mathbf{n_{ou}}}{\mathbf{2}}$-edges between inner vertices in (top and bottom)

Main results:

Random hyper tangle graph:

Let $(\mathbf{T_h})_{\mathbf{n\,M}}$ be hyper tangle graph with n vertices and M edges**,** where n consisting of inner and outer vertices.

The number of possible edges is given by:
$\binom{n_{in}}{k}+\binom{\mathbf{n_{ou}}}{\mathbf{2}}$-edges between inner vertices in (top and bottom) =R where k size of hyper graph (k- uniform tangle hyper graph).

All possible graphs $\mathbf{C_{nM}}$ are given by:

$$\binom{R}{M}$$

This called possible choices.

If $\mathbf{G_{nM}}$ denotes any one of $\mathbf{C_{nM}}$ graphs, then:
P (T_n,M) is identical with $\mathbf{G_{nM}} = 1 \ / \ \boldsymbol{C_{nM}}$

Corollary:

If some graphs from $\mathbf{G_{nM}}$ provide with property say (A) then:
The probability of random hyper tangle graph $\mathbf{(T_h)_{n\,M}}$ is given by:

$$\mathbf{P_{nM}\,(A)} = \boldsymbol{A_{nM/C_{nM}}}$$

Where $\boldsymbol{A_{nM}}$ is the number of those $\mathbf{G_{nM}}$ with have property (**A**).

Distribution function:

$$\mathbf{P}\,[\mathbf{deg}(\mathbf{v})=\mathbf{k}]=\binom{\mathbf{n}-\mathbf{1}}{\mathbf{k}}\boldsymbol{p}^{\,\mathbf{k}}\,(\mathbf{1}-\boldsymbol{p})^{\mathbf{n}-\mathbf{1}-\mathbf{k}}\,\Big|\,\mathbf{ou}+\binom{\mathbf{n}-\mathbf{1}}{\mathbf{k}}\boldsymbol{p}^{\,\mathbf{k}}\,(\mathbf{1}-\boldsymbol{p})^{\mathbf{n}-\mathbf{1}-\mathbf{k}}\,\Big|\,\boldsymbol{in}$$

Degree of tangle hyper graph:

If the tangle hyper graph (T_h) has (n) vertices (inner- outer) and (N) edges, we call the number (Z) the degree of the tangle hyper graph.

$$Z=2\left(\frac{\binom{n_{in}}{k}+\binom{\mathbf{n_{ou}}}{2}-\text{edges between inner vertices in (top and bottom)}}{n\ in}+\frac{\binom{n_{in}}{k}+\binom{\mathbf{n_{ou}}}{2}-\text{edges between inner vertices in (top and bottom)}}{n\ out}\right)$$

Balanced tangle hyper graph:

If the tangle hyper graph (T_h) has property that has no sub graph having larger degree than tangle hyper graph, we call (T_h) balanced tangle hyper graph.

Complete tangle hyper graph:

Agraph is called complete tangle hyper graph $\left(K_{T_h}\right)_M^k$ of order $\binom{k_1}{M}+\binom{k_2}{M}$ Where (k_1 ,k_2) inner and outer vertices respectively, and $\binom{k_1}{M}+\binom{k_2}{M}$ are possible edges. Thus in a complete tangle hyper graph, if every vertices is adjacent.

Regular tangle hyper graph:

A graph is called K- regular tangle hyper graph if every vertex $x \epsilon v$ has degree K.

Complementary tangle hyper graph:

$\overline{(\text{Th})}$ A graph $\overline{(\text{Th})}$ is called complementary tangle hyper graph if

Consists of the same vertices, as T_h and of those and only those edges which do not occur in T_h .

References

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