Estimation of the Confidence Intervals for the Average Loss Function of Taguchi and Signal to Noise ratio through Resampling Bootstrap Method

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Abstract

The constant improvement in the manufacture of products is a way for these companies stay competitive in the market, and for this to be possible, investments in quality become essential. Such improvements aim for the production of more uniform products and robust in front of functional variations. To measure the uniformity and robustness of those products is necessary the use of appropriate tools, being these tools (quality indicators) the average loss function of Taguchi and the signal to noise ratio. Through the average loss function of Taguchi it can be measured the average loss, in monetary units, due to functional variation of a variable answer regarding its project (target), being this quality indicator used to justify investments in quality. The signal to noise ratio assists, in the project phase, in the fixation of values of parameters with the intention of minimizing the effect of noises (undesirable and uncontrollable factors) in the performance of some characteristic of the product, synthesizing in a single value the relationship between the wanted sign and the unwanted (noise). This way, such indicators justify the importance of investments in quality searching for, like this, centered productive processes and uniforms. Due to its contribution in the area of quality engineering, it becomes interesting the accomplishment of statistical inferences in the two quality indicators mentioned, just as the estimate of confidence intervals. As a classic statistical method does not exist to estimate of these intervals, is proposed to do that, their estimates are done through the resembling method computationally intensive, due to the generality of its application and reliability of the results.

Keywords: Loss Function of Taguchi. Signal to noise ratio. Estimation. Confidence Intervals. *Bootstrap*

1. *Introduction*

The quality engineering is a branch of the engineering developed specifically for the development and application of scientific methods with the intention of improving the quality of a product (service) constantly. In agreement with Ribeiro and Elsayed (1993, apud Caten 1995), the objectives of the quality engineering are:

1) To minimize the deviations of the value target;
2) To maximize the robustness to the noise;
3) To maximize the robustness to possible oscillations in controllable factors.

So for the execution of the mentioned objectives, it should be analyzed the measurable characteristics of a product with the maintenance intention and constant improvement of its quality. The analysis, with an objective of improvement of the quality, utilize of statistical methods to diagnose, to monitor, to model and to improve productive processes. According to Chaves Neto (2013), the statistical techniques can be useful in every production cycle, because it is widely used in the quantification of the variability of the process, in the analysis of this variability in relation to the specifications of the product and, in the support of the administration in the elimination or reduction of that variability.

In general, the statistical analysis and estimate of some measured wanted is accomplished through samples, even if the same ones are representative of the population, and such estimate can be accomplished by some statistical methods, such as: the method of the maximum likelihood, the method of the moments, the least square method and for simulation computational. The chosen method to estimate of the wanted parameters is the simulation, because through this can take place statistical inferences regarding any measured of uncertainty. The deduction of relative information to a population, by the use of random samples of it extracted, concerns the statistical inference (MARQUES; MARQUES 2009). The chosen simulation method is the bootstrap, a computationally intensive method that uses the resembling from the original sample (master sample) to estimate some measure of interest. Bradley Efron introduced the method bootstrap in 1979. The justification for the great utilization and success of this statistical method of resembling designated by the existence of solutions based on asymptotic considerations for several problems, in such way that is possible to apply the method bootstrap in situations where few sampling observations exist or that depend on the supposition of normality. In this context, a question comes up: Why or when should it use the bootstrap method? According to Schmidheiny (2012) it should be used it by two main reasons:

1ª) when the asymptotic distribution is very complicated of obtaining;

2ª) for the fact of the bootstrap to obtain "better" approaches for certain properties.

In the second case, it can be demonstrated that the approaches that use the bootstrap method converge more quickly to certain statistics than the approaches done by the asymptotic theories. Those approaches, used of the bootstrap method are denominated of asymptotic refinements. With regard to the statistical inference can be mentioned "The bootstrap method offers a very powerful solution for statistical inference when the traditional statistics cannot be applied" (DUTANG et al.2008). This way of context, given a functional characteristic of a product (service), the bootstrap method it can be utilized to estimate of the confidence intervals of the average loss function of Taguchi and signal to noise ratio, because the same is not made nowadays. In this article, is showing the estimate of three types of confidence intervals using the bootstrap method for the quality indicators mentioned.

2. Average loss function of Taguchi

To invest in quality inside of a production process, it becomes necessary to justify it and knowing the costs involved in it. Inside of this context, the companies classify the costs with quality in three categories:

1) Costs with prevention that concerning the maintenance of the quality; system; 2) Evaluation costs, regarding the costs for maintenance of the system of quality warranty; 3) Costs with internal and external failure that they correspond at the costs regarding production losses, refuse, rework, warranty of the product, repair of the product among others.

The knowledge of the costs involved in a productive process is of extreme importance for the industries, because it can only justify investments like this in quality. With that, the questions appear. How much will cost the absence of the quality? Is it possible to measure that cost? Those two questions it can be answered through the loss function of Taguchi, where the same express the monetary losses originating from of the deviations of the variable answer, in relation to some functional characteristic of the product, to the project target value. Such deviations take a loss for the consumer, for the manufacturer of the product and for the society in a long term. Taguchi, apud Nakagawa (1993), it developed a method that allows to quantify the impact of the bad quality, in monetary units, through the loss function. This way it gets to measure the impact of the losses in a product not only for the consumer, but also for society in long period. According to the philosophy of Taguchi, the loss for the society is composed of costs that can be identified in a direct way and in an "indirect" way. The costs that can be identified in a direct way in regards to those that are verified in the production process of a product, or in the execution of a service, that involves from expense with raw materials, work hand, machinery, scrapping, product replacement, replacement of some badly service and so on.

The costs "indirect" or subjective are those that it does not get to see in a direct way the monetary value involved in a process of low quality and they are usually related to the customer's dissatisfaction, since the monetary quantification of the losses will only be directly identified, when the product or service to lose market for their competitors. This way, through the loss function, it is demonstrated that high quality is free of costs associated with lower quality.

Such statement is verified thoroughly in the Japanese and American industry, which through the loss function has to quantify the benefits obtained by the reduction of variability of a characteristic of the product around its target value. Therefore, such function measures the quality and presuppose that the objective is to produce items, under a controlled process, in such a way that the consumer's expectations are assisted with to the smallest possible variability of some characteristic of it, taking into account the suffered damage for the society in a general way.

"Taguchi affirms that the loss for the society, as a whole, is minimum when the performance of the product assists to the nominal value (wanted performance) and the quantification of this loss in monetary value it does with that all understand the importance of the improvement of the process of production of the product (service), from the simplest worker to the president of the company" (CHAVES NETO 2013). In agreement with Ealey (1988), Taguchi found the quadratic representation of that function and it demonstrated that the same is efficient and that it allows making a realistic estimate for low quality (deviation of the nominal value μ). The representation of that function has variations in agreement with the type of analyzed functional characteristic. When the functional characteristic is of the nominal-the-better type, the variable answer $y \in R$ and the loss function is defined in agreement with the equation (1).

$$\mathrm{L}(\mathrm{y}) = \mathrm{k}(\mathrm{y} - \mu)^2 \tag{1}$$

When the functional characteristic is of the smaller-the-better type, the variable answer $y \in R^+$ and its nominal value is equal to zero. The loss function for that characteristic is defined by the equation (2).

$$\mathrm{L}(\mathrm{y}) = \mathrm{ky}^2 \tag{2}$$

If the variable answer $y \in R^+$ and its nominal value are the largest possible for a better performance of the product, the large-the-better functional characteristic is getting, whose function is defined by the equation (3).

$$\mathrm{L}(\mathrm{y}) = \mathrm{k}\left(\frac{1}{\mathrm{y}^2}\right) \tag{3}$$

Frequently the manufacturers need to evaluate the average quality of its product, in relation to a characteristic of it, along a period of time. This can be measured through the development of the average loss function for *n* units of a product (service). The method to estimate the average loss for *n* products in the nominal-the-better case, it is developed being used the arithmetic average of $(\mathrm{y} - \mu)^2$, that is denominated of MSD (Mean Square Deviation). This way, it is get that

$$MSD = \frac{(y_1 - \mu)^2 + (y_2 - \mu)^2 + \cdots + (y_n - \mu)^2}{n} \tag{4}$$

or

$$MSD = \frac{\sum_{i=1}^{n} (y_i - \mu)^2}{n} \tag{5}$$

It is demonstrated that an equivalent form for writing (5) is

$$MSD = \frac{(y_1 - \bar{y})^2 + (y_2 - \bar{y})^2 + \cdots + (y_N - \bar{y})^2}{n} + (\bar{y} - \mu)^2 \tag{6}$$

or

$$MSD = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n} + (\bar{y} - \mu)^2 \tag{7}$$

Like this, get that $\bar{y}$ is the arithmetic average of the functional y characteristic defined for

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \tag{8}$$

and the variance around the average $\bar{y}$ is given for

$$\sigma^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n} \tag{9}$$

This way, it can be rewritten (7) as

$$MSD = \sigma^2 + (\bar{y} - \mu)^2 \tag{10}$$

The value of the Mean Square Deviation (MSD) in (10) it represents a measure of the variance (variability around the average) and the deviation of the average value in relation to the nominal value μ .

This way, the average loss function, defined for $\bar{L}(y)$ it can be written as

$$\bar{L}(y) = k(MSD) \tag{11}$$

Or

$$\bar{L}(y) = k\left[\sigma^2 + (\bar{y} - \mu)^2\right] \tag{12}$$

In a similar way to deduce the equation (12), through the Mean Square Deviation (MSD), it extends the concept that average loss function of Taguchi for the functional characteristics of the smaller-the-better and large-the-larger type, where the same ones are given by the equations (13) and (14) respectively.

$$\bar{L}(y) = k\left[\sigma^2 + \bar{y}^2\right] \tag{13}$$

$$\bar{L}(y) = k\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right] \tag{14}$$

3. Signal to noise ratio

The signal to noise ratio assist, in the project phase, in the fixation of values of parameters with the intention of minimizing the effect of noises (undesirable and uncontrollable factors) in the performance of some product characteristic, synthesizing in a single value the relationship between the wanted sign and the unwanted (noise). The evaluation of performance of a product is an important part of the process of measurement of the quality of it, this way, it becomes imperative to develop a metric efficient for evaluation of performance of a product. This metric should take into account two types of output of a product manufacturing system, the exit of the system that it wants to reach and the exit of the system that is undesirable. This way the metric should include the desirable and undesirable aspects of performance simultaneously in a single value. With that, it becomes important to study the effect of the variation of some characteristic of the product (service) with the intention of verifying its influence in the performance of it. The quality indicator with the desirable characteristics to measure the robustness of a product (service), according to the functional variations, it is denominated of signal to noise ratio and it was idealized by Genichi Taguchi. Taguchi et al. (2004) idealized a transformation of the data in the repetition of an experiment, in such a way that transformation represents the measure of the existent variation and its influence in some functional characteristic of the product or process. This transformation is denominated of signal to noise ratio (S/N) and this relationship combines several repetitions with the intention of verifying how much of variation it is present in the performance of the product.

The signal to noise ratio, measures the magnitude of the true information (sign) after some uncontrollable variations (noise), and depending on the measured functional characteristic, an equation of signal to noise ratio exists adapted for it. As measure, it reflects the variability of the answer of a system caused by factors of noises, independently of adjustments in the average, in other words, it is useful to predict the quality even if the projected nominal value changes, and also in this case, to be used for comparison of proposals.

The procedure for deduction of the equations of the signal to noise ratio is based on the Mean Square Deviation (MSD) of the average loss function of quality, which is calculated through dispersion measures, just like the variance. When the variable answer follows the nominal-the-better characteristic, it has two types of signal to noise ratio, NTB type I and NTB type II, given respectively by the equations (15) and (16).

$$S/N_{tipo\ I-NTB} = 10\log\left[\frac{\bar{y}^2}{s^2}\right] \tag{15}$$

and

$$S/N_{tipo\ II-NTB} = -10\log\left[s^2\right] \tag{16}$$

The equation (15) it is used when the variable answers $y \in R^+$, and the equation (16) it is used when the variable answer $y \in R$, and both variables are of the nominal-the-better type.

The s^2 value is given by the calculation of the variance in relation to the average of the answer variable y, in other words, it measures the dispersion of the answer in relation to $\bar{y}$. When the characteristic of the variable answer is of the smaller-the-better type it is had that, the signal to noise ratio is given for

$$S/N_{STB} = -10\log\left[\frac{1}{n}\sum_{i=1}^{n} y_i^2\right] \tag{17}$$

and when, the variable answer is of the large-the-better type, it is had that the signal to noise ratio is given for

$$S/N_{LTB} = -10\log\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right] \tag{18}$$

In both cases, given by the equations (17) and (18), the variable answer $y \in R^+$, being that the scale used for the signal to noise ratio views is the decibel (dB).

4. The resample bootstrap method

Depending on the accomplished experiment, the estimate of a certain uncertainty can be complicated and even very extensive of obtaining, in other words, it can get the potentially wrong results when it is made by hypotheses or simplifications inappropriate for a certain statistical model. According to Babu (2011), the statistical classic methods concentrate mainly on the statistical properties of estimators that has a form relatively simple and that they can be analyzed mathematically, and the methods involved in its analysis, many timidities used of simplifications and presuppositions non- realists. According to Hesterberg (2011) some ideas in statistics and in probability are very complex for students, including the sample distributions, and in this extent the computational simulation, makes that the same ones win in the intuitive sense of those concepts. In this context, there are statistical measures whose estimates and inferences are simple of obtaining in a clear way, such as the arithmetic mean and the standard deviation. However, for measures more complex do not exist formulas for the accomplishment of inferences about it or, when they exist, it is very complex to be used by the traditional ways in statistics. This way, the computational simulation is an alternative way for the accomplishment of inferences regarding some measured of uncertainty and the used simulation method in this article, for the calculation of confidence intervals of the average loss function of Taguchi and signal to noise ratio is denominated of bootstrap.

The bootstrap is a computationally intensive resampling method, introduced in 1979 by Bradley Efron, whose advantages are of more easy understanding and generality of application, because the same one require that a few suppositions are made and its answers are more accurate than other resamples methods. The central proposal of the method bootstrap is to demonstrate through the computational use the obtaining of standard errors, confidence intervals, the estimate probability distribution and the calculation of another measured of statistical uncertainties for a variety of problems. "The bootstrap is a statistical technique related to the jackknife, introduced by Efron in 1979 and it is shown versatility in the treatment of a variety of statistical problems" (CHAVES NETO 1986).

In agreement with Davison and Hinkley (1997), the key idea of the bootstrap is the resample of the original data to create a group of replicated data, and starting from them, the variability of the amounts of interest can be evaluated without an extensive and prone analytical calculation to mistakes. Resample or replicate, in this case, is to randomly select a new sample, of a sample already known, with replacement of the group of original data.

The method bootstrap is very used when the sample goes very small or when the calculation of estimators and accomplishment of inferences about it are complicated or even impossible of obtaining. To accomplish the estimate of a parameter for the bootstrap it is necessary a large number of samplings, and for such it is necessary the support of fast computers and computational programs. According to Davison and Hinkley (1997) "as this approach is to repeat the procedure of data analysis with many replicated groups, this procedure is computationally intensive named." So the method is widely used for estimation of sampling distributions. The main objective in the use of the bootstrap as resample technique is to demonstrate, through the computer, data recovery to obtain a reliable standard error, reliable confidence intervals and other reliable statistical measures for a range of problems. In the use of the method, in a simplified way, is made the resample through the experiment's collected data. With that, several problems in statistics can be avoided, such as excessive simplifications in problems more complex. To begin with, the method bootstrap, a master sample size n, that is representative of the population, $\underline{x}' = [x_1, x_2, \ldots, x_n]$ should be removed. The master sample is defined, also, as original sample of the population that generated it. For such, this sample should be picked in a planning way for do not commit calculation mistakes and with that to obtain reliable results.

The most part of authors recommend the use of at least 1000 resample, where those B samples bootstrap (resample) should have the same size the sample master n, $\underline{x}_1^*, \underline{x}_2^*, \ldots, \underline{x}_B^*$ such that the chosen values starting from the master sample should be done in a random way. In view of given resample, the statistics $\hat{\theta}_i^*$ should be calculated for each one of them. This way, according to Bussab and Morettin (2012) "the basic idea of the bootstrap is resample the available group of data to estimate the parameter θ, in order to create replicated data. Starting from those replications the variability of an estimator can be proposed for θ, without appealing to analytical calculations." The superscript "*" indicates that the calculation of a statistic is starting from a sample bootstrap and the statistics of master sample will be represented for $\hat{\theta}$.

5. Bootstrap confidence intervals

In agreement with Bennett (2009), the definition to proceed is removed of The Cambridge Dictionary of Statistics (Everitt 1998):

A confidence interval is a range of values, where calculations begin from the observations of the sample, with a known probability of containing the true value of the parameter. A confidence interval of 95%, for example, indicates that this estimate process was repeated several times, until the expected of 95% from calculated intervals contain the real value of the parameter. It is verified that the confidence level (known probability of the interval) refers to the properties of the interval and not for the parameter itself, that is a random variable considered. The confidence intervals based on the method bootstrap are very used when one does not want to make suppositions associated to the classic statistic, or when it is not having the statistics of interest distribution explicitly. It is known, also, that the application of the bootstrap is very useful and effective, when the size of the sample is small. This way, the three types of confidence intervals proposed is defined, based on the method bootstrap: 1) standard bootstrap confidence interval; 2) bootstrap confidence interval t; 3) confidence interval percentile. The standard bootstrap confidence interval for a statistic of interest and with a covering probability $(1-2\alpha)$ it is defined as

$$IC_{bootpadrão} = \left(estatística \pm z_{\alpha} \times SE_{boot} \right) \tag{19}$$

Where z_{α} is $\alpha - ésimo$ the value of the standard normal distribution of scoresand SE_{boot} is the bootstrap standard deviation (standard error) defined by the equation (23).

The interval of trust bootstrap t for a statistic is defined for

$$IC_{bootstrap\ t} = \left(estatística \pm t_{df} \times SE_{boot} \right) \tag{20}$$

where n is the sample master's size, *df* are the degrees of freedom of the score t, being this equal to the (n -1), t_{df} is the value of t the score with df degrees of freedom, SE_{boot} is the bootstrap standard deviation (standard error) defined in (23).

According to Rizzo and Cymrot (2006), the percentile confidence interval can be calculated in two ways:

(1) To find the percentile $\left(1-\alpha/2\right)100\%$ and the percentile $\left(\alpha/2\right)100\%$ of the average of the statistic´s resamples of the parameter that it wants to estimate;

(2) To obtain the interval of percentile trust, through the percentile of the differences of the statistic's values of the resamples in relation to this same statistic´s average value in the resamples. The method adopted in this article for the calculation of the interval of percentile trust will be the first method mentioned.

For a confidence level of 95%, calculations are made the percentile 2.5% and 97.5% of all estimates of interest B bootstrap samples, and it is finding the confidence interval of percentile thus

$$IC_{boot\ percentil} = \left[P_{2,5\%}\left(\hat{\theta}^*\right); P_{97,5\%}\left(\hat{\theta}^*\right)\right] \tag{21}$$

In a similar way, for a confidence level of 99%, calculations are made the percentile 0.5% and 99.5% of all estimates of interest B bootstrap samples, and the confidence interval is defined for

$$IC_{boot\ percentil} = \left[P_{0,5\%}\left(\hat{\theta}^*\right); P_{99,5\%}\left(\hat{\theta}^*\right)\right] \tag{22}$$

Once calculated the confidence interval of percentile it may, through this, verify that the bootstrap confidence interval t is reliable. Therefore, it is necessary that both intervals have small addictions, with very close values.
As much for the calculation of the interval of trust standard bootstrap, as for the interval of trust bootstrap t, is used of the calculation of the standard deviation bootstrap (standard error) defined for

$$SE_{boot} = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}\left(\hat{\theta}_i^* - \frac{1}{B}\sum_{i=1}^{B}\hat{\theta}_i^*\right)^2} \tag{23}$$

Another important definition is regarding the addiction of the estimate of a parameter. In agreement with Montgomery and Runger (2003), the statistic used to estimate a parameter is addicted when its sample distribution it is not centered on the true value of the parameter. The estimate of the bootstrap addiction can be verified and for the same one is made calculations, comparing the bootstrap distribution of the statistic in questionis centered in the sample master's statistic. This way the estimator of the addiction, it is defined for:

$$vício_{boot} = \overline{\hat{\theta}^*} - \hat{\theta} \tag{24}$$

The calculation of the addiction is essential to verifying of the validity of the confidence intervals obtained by the bootstrap method. According to Borkowski (2013, apud LOIBEL AND ALVES 2013) it may be considered the small addiction if the same is smaller than 25% of the standard deviation of the addiction that is defined for

$$DP_{vício} = \sqrt{\sum_{i=1}^{B}\frac{\left(\hat{\theta}_i^* - \hat{\theta}\right)^2}{B-1}} \tag{25}$$

This way, in agreement with the equation (25) it may be verified if the estimates of the confidence intervals previously mentioned are reliable.

6. Material and methods

The master sample (original sample) is referring the type of large-the-better functional characteristic. The same one it was remove from the book of *Engenharia de Qualidade em Sistemas de Produção*, because it is complicated to estimate the loss for the consumer in a practical case because involves indirect costs, therefore subjective. The functional characteristic in question is the resistance of a sticker, where the same one is determinate by the necessary force to separate the linked items by the sticker. The inferior limit of the specification (functional tolerance) is of 5 kgf for the rupture load; where the items out of the specification are scrap resulting in a loss for the consumer of $70.00 per item. The samples are referring the measurement of the rupture force in two types of stickers S_1 and S_2.

Table 1: Data of the force of rupture of stickers S_1 and S_2

TYPE OF STICKER	FORCE OF RUPTURE (kgf)							
S_1	10,2	5,8	4,9	16,1	15,0	9,4	4,8	10,1
	14,6	19,7	5,0	4,7	16,8	4,5	4,0	16,5
S_2	7,6	13,7	7,0	12,8	11,8	13,7	14,8	10,4
	7,0	10,1	6,8	10,0	8,6	11,2	8,3	10,6

Source :TAGUCHI, G. et al.,1990, p. 48

In this specific case, where the characteristic in question is of the large-the-better type, the value of k (quality loss coefficient) will be about 1750. The objective of using two samples is to compare the quality of the stickers in relation to the estimates of the quality indicators, average loss function of Taguchi and signal to noise ratio.
The average loss function of the master´s sample described is defined for

$$\bar{L}(y) = 1750\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right] \tag{26}$$

and the same will be calculated for each bootstrap sample. This way, it is had that

$$\bar{L}_i^*(y) = 1750\left[\frac{1}{n}\sum_{j=1}^{n}\frac{1}{y_{ij}^2}\right] \tag{27}$$

where $i = 1,\ldots,B$ .This way it is had $\hat{\bar{L}}_1^*(y), \hat{\bar{L}}_2^*(y), \ldots, \hat{\bar{L}}_B^*(y)$, and starting from these estimates, it is calculated the variability and the confidence intervals of the average loss function of Taguchi for the large-the-better functional characteristic.

The signal to noise ratio of the sample´s master described is defined for

$$S/N_{LTB} = -10\log\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right] \tag{28}$$

And the same will be calculated for each bootstrap sample. This way, it is had that

$$S/N_{i\ LTB}^* = -10\log\left[\frac{1}{n}\sum_{j=1}^{n}\frac{1}{y_{ij}^2}\right] \tag{29}$$

Where $i = 1,\ldots,B$. This way it is had $S/N_1^*, S/N_2^*, \ldots, S/N_B^*$, and starting from these estimates, it is calculated the variability and the confidence intervals of the signal to noise ratio for the large-the-better functional characteristic.
The whole development of the algorithm, as well as the proposed calculations, it was performed in the software named Matlab.

7. Results and analysis

In the table 2, it is observed that the master sample S_2 has a smaller average loss than the master sample S_1, because observing the sample, it is verified that is necessary to do more force to separate the attached items for the sticky tape of the sample S_2.

Table 2: Results for the average loss function for the case LTB

CALCULATIONS	S_1			S_2		
	B			B		
	10000	20000	30000	10000	20000	30000
$\bar{L}(y)$	39,9710	39,9710	39,9710	19,9302	19,9302	19,9302
SE_{boot}	8,8955	8,9158	8,8974	2,4688	2,4674	2,4723

Source: The author

The results refer to signal to noise ratio, synthesized in the table 3, it demonstrates a smaller value for the signal to noise ratio of the sample S_1 in comparison with S_2 .

Table3: Results for the signal to noise ratio for the case LTB

CALCULATIONS	S_1			S_2		
	B			B		
	10000	20000	30000	10000	20000	30000
S/N (dB)	16,4129	16,4129	16,4129	19,4353	19,4353	19,4353
SE_{boot} (dB)	1,0241	1,0215	1,0174	0,5401	0,5432	0,5428

Source: The author

According to the results, regarding the variability calculated by the bootstrap method, it is observed that in the tables 2 and 3 are small variability between the results of 10000, 20000 and 30000 bootstrap samples for each master sample. This way, it is concluded that 10000 bootstrap samples are enough to estimate of confidence intervals of the proposed statistics. The results of the calculations of the confidence intervals proposed in this article are synthesized in the tables 4 and 5. The table 4 demonstrates the three confidence| intervals of 95% and 99% proposed, and verified, a small addiction in relation to the two master's samples.

Table 4: Confidence intervals of the loss function of Taguchi for the case LTB with 10000 samples bootstrap

TYPES OF INTERVALS OF TRUST	LEVEL OF TRUST	S1		S2	
		LI	LS	LI	LS
Bootstrap Standard	95%	22,5361	57,4059	15,0282	24,8323
Bootstrap t		21,0107	58,9314	14,5993	25,2612
Bootstrap Percentile		23,1158	57,5103	15,2878	25,0927
Bootstrap Standard	99%	17,0577	62,8843	13,4878	26,3727
Bootstrap t		13,7585	66,1835	12,5602	27,3003
Bootstrap Percentile		18,2525	63,3114	14,1157	26,8109
Addiction		Small		Small	

Source: The author

The table 5 demonstrate the results of the three trust intervals proposed for the signal to noise ratio of the large-the-better type, and in both samples, the addiction was considered small.

Table 5: Confidence intervals for the signal to noise to ratio for the case LTB with 10000 samples bootstrap

TYPES OF INTERVALS OF TRUST	LEVEL OF TRUST	S1		S2	
		LI	LS	LI	LS
Bootstrap Standard	95%	14,4057	18,4201	18,3621	20,5084
Bootstrap t		14,2301	18,5958	18,2682	20,6023
Bootstrap Percentile		14,8329	18,7913	18,4349	20,5869
Bootstrap Standard	99%	13,4201	19,0509	18,0249	20,8456
Bootstrap t		13,3952	19,4307	17,8218	21,0487
Bootstrap Percentile		14,4156	19,8172	18,1473	20,9334
Addiction		Small		Small	

Source: The author

According to the results of the tables 4 and 5, it is verified that it may use any one of the three types of trust intervals proposed, because its values are close and its addictions are small.

8. Conclusion

According to the obtained results, it is verified that the sticker S1 presents a larger average loss than the sticker S2, however a smaller signal to noise ratio, what takes to conclude that the sticker S2 has a superior quality in relation to the sticker S1 in relation to the indicators of quality loss function of Taguchi and signal to noise ratio. Besides comparing the samples regarding the quality indicators proposed could be estimated the confidence intervals of 95% and 99% for the parameters average loss of Taguchi and signal to noise ratio through the bootstrap method. The method proved to be efficient, because even not knowing the distribution that generated the sample it was possible to achieve the proposed goals, and relatively simple to apply, therefore it was not necessary to do any supposition or simplification in the calculations of the estimates of the confidence intervals of the quality indicators proposed. According to the obtained results, it was observed that the use of any one of the three types of proposed confidence intervals is viable, since all have similar values and small addictions. Due to the effectiveness of the application of the bootstrap method to estimate of the confidence intervals proposed, it can incorporate more two types of confidence intervals, denominated of BC and BCa, used when it is had a big addiction and asymmetry presents. In addition, it can apply the bootstrap method to estimate of the distributions of probability of the indexes C_p, C_{pk} and C_{pkm} with its respective confidence intervals for any one of the three existing functional characteristics.

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