On the Conversion of Coordinates from Nigeria Transverse Mercator to Universal Transverse Mercator Using a Simple Mathematical Model

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Abstract

In this paper, effort has been made to transform coordinates from Nigerian Transverse Mercator (NTM) system to Universal Transverse Mercator (UTM) system using a Simple Mathematical Model. The data used are the simulated UTM coordinates of points in the Mid Belt of UTM projection system which also fall in zone 32 of the UTM projection system, which also covers Rivers State of Nigeria. The equivalent NTM coordinates of these points were obtained using the Defense Mapping Agency (DMA) Multiple Regression Equations as programmed in Geographic Calculator software. An orthophoto imagery used by the Rivers State Government Geographic Information System (RIVGIS) to chart surveys in the State was used to overlay the results obtained by the Model All the computations were carried out using Microsoft excel program. Based on the results obtained by the Mathematical Model, it can be inferred that the Model is an alternative tool for coordinate transformation from NTM system to UTM system. The analysis further revealed that the error does not follow a progressive pattern, which indicates that the Model is not good enough for geodetic or engineering purposes. However, the results show that the Model is significant for plane surveys where calculating land composition and topography considers a set expanse of land as a flat plane.

Keywords: Coordinate transformation, Projection System, Ellipsoid. Mathematical Model, Multiple Regression technique, Orthophoto, NTM and UTM

Introduction

The River's State Government of Nigeria in 2005 acquired an orthophoto (Aerial Photograph) map for Rivers State Government Geographic Information System (RIVGIS). One of the aims of this was to ensure that all surveys and mapping in the state are done with a reasonable degree of accuracy before title registrations. The coordinate system used for the orthophoto map was World Geodetic System 1984 (WGS84) Universal Transverse Mercator (UTM**)** Zone 32**.** Consequently, the River's State Government introduced WGS84 UTM Zone 32N coordinate system in 2011 for all surveys and mapping. This was to replace the Nigerian Transverse Mercator (NTM) with reference to Mid Belt Clark 1880 Ellipsoid, which had been in use. For revalidation and map revision, this, therefore, necessitated the transformation of coordinates of points in NTM system to the equivalent coordinates of those points in UTM system.

Previous attempts made by Edoga (1979), Karney (2011), and Idowu (2012) among others using analytical or numerical techniques to transform coordinates from NTM to UTM has been successful.

However, the techniques involved are laborious, rigorous and difficult to understand in approach, and the coordinates used for such transformation were from the West Belt of NTM system, which also falls in Zone 31 of UTM system. Didigwu (2005) presented a technique that is simple in approach yet gives satisfactory results for points within a few kilometers. His work was limited to the general mathematical formulae provided for the computation of three variation factors for scale, rotation of axis and origin. Therefore, it is the objective of this paper to apply Didigwu (2005) technique to convert coordinate of points in NTM to the equivalent coordinates of those points in UTM system.

Transverse Mercator, Nigerian Transverse Mercator and Universal Transverse Mercator

Maps are basically a flat or planar representation of part or the earth's entire surface. The basic problem of map making is that it is impossible to develop a surface with double curvature, such as a sphere or ellipsoid, onto a plane surface without distortion of some kind (Gregory, 1982). Different map projections are designed to maintain some property of the ellipsoidal surface undistorted. These include among others, azimutal, oblique, cylindrical and conformal projections.

The Transverse Mercator (TM) is a conformal cylindrical projection and may be visualized as a cylinder wrapped around the earth and oriented so that its axis is in the plane of the equator. The cylinder is often slightly smaller than the earth in radius and intersects it along two ellipses equally spaced from and parallel to a central meridian of longitude (Uzodinma and Ezenwere 1993). When the cylinder is developed onto a plane, the meridians (longitude) and parallels (latitude) intersect at right angles (Figure 1). The central meridian is a straight line and the nearby meridians are slightly concave (nearly straight lines) to the central meridian. The parallels are curved lines concave to the nearest pole (Gregory, 1982).

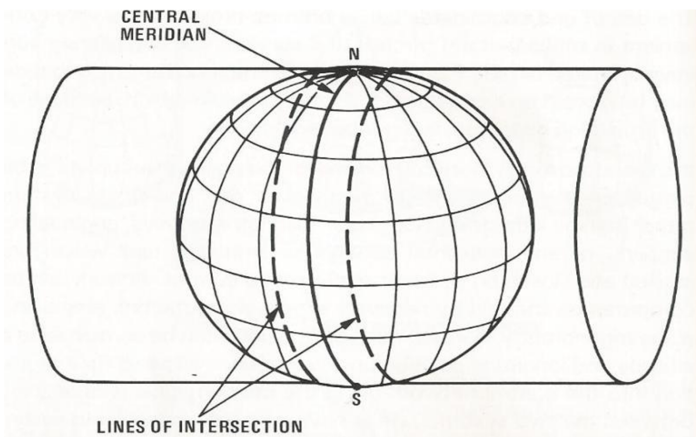

Figure 1: The Transverse Mercator projection (Gregory, 1982)

NTM is a modified version of TM adopted for Nigeria. The modifications take care of the large expense of the country which covers latitudes 4^o 15'N to 14^o 15'and longitudes 2^o 30'E to 14^0 30'E. It is generally divided into three belts: west, mid and east belts respectively. Each belt is 4^o wide in longitude and uses a central scale factor of 0.99975. The UTM which is the universally accepted projection system is also based on TM with more modifications to the TM (Idowu, 2012). Its application is limited to between latitudes 84^o N and 80^o S. The polar areas are covered by the Universal Polar Stereographic projection (UPS). The UTM overlaps 30' onto the UPS, which extends from the poles to 83^o 30'N or 79^o 30'S respectively (DMATM 8358.2, 1989). The UTM has zones 6^o wide in longitude and uses a central scale factor of 0.9996.

Transformation of Coordinates from NTM to UTM

Coordinate transformation can be defined as the process of establishing the relationship between coordinates systems in order to convert points from one system to the other. There are various methods of coordinate transformation: - projective, affine, conformal transformations etc. The choice of method depends on the purpose for which the transformation is designed (Idowu, 2012). In this paper, the transformation parameters were derived using a simple mathematical model. This has the property of preserving small shape during transformation. Hence, coordinates of NTM points were transformed to their equivalent UTM.

A Simple Mathematical Model for Coordinate Transformation

Suppose the coordinates of point **A** in NTM system are (***e***, ***n***), the coordinates of **A** in UTM system are (**E**, **N**) and the deviation in bearing from **A** to a point **B** between the two systems is **λ**, then the formulae for the three variation factors of scale, rotation of axis and origin can be determine as follows (Didigwu, 2005):

$$\mathbf{E} = \pounds + k\boldsymbol{n}\mathrm{Sin}\lambda + k\boldsymbol{e}\mathrm{Cos}\lambda \quad (1a)$$
$$\mathbf{N} = \mathcal{N} + k\boldsymbol{n}\mathrm{Cos}\lambda - k\boldsymbol{e}\mathrm{Sin}\lambda \quad (1b)$$

Where k is scale factor, λ is the deviation in bearing, £ and $\mathcal{N}$ are origin shifts in the easting and nothing axes respectively, and cosλ and sinλ are rotation of axes.

The scale factor (k) is given by:

$$k = 1 + (k_{UTM} - k_{NTM}) \quad (2)$$

The numerical values of k_{UTM} and k_{NTM} as provided by ESRI (2008) are 0.9996 and 0.99975 respectively. Substituting these values in equation (2) gives k as 0.99985,

The deviation in bearing (**λ**) is given as:

$$\lambda = \beta_2 - \beta_1 \quad (3)$$

β_1 and β_2 are determined as:

$$\beta_1 = \tan^{-1}(\Delta e/\Delta n) \quad (4a)$$
$$\beta_2 = \tan^{-1}(\Delta E/\Delta N) \quad (4b)$$

Where β_1 is the bearing from the Mid Belt origin to the UTM Zone 32 origin in NTM system and β_2 is the bearing from the Mid Belt origin to the UTM Zone 32 origin in UTM system.

Let

$$\alpha = k\cos\lambda \quad (5a)$$
$$\gamma = k\sin\lambda \quad (5b)$$

Equations (1a) and 1(b) now becomes

$$\mathbf{E} = \pounds + \gamma\boldsymbol{n} + \boldsymbol{\alpha e} \quad (6a)$$
$$\mathbf{N} = \mathcal{N} + \boldsymbol{\alpha n} - \boldsymbol{\gamma e} \quad (6b)$$

From equations (6a) and (6b), the origin shifts (£ and $\mathcal{N}$) can therefore be presented as:

$$\pounds = \mathbf{E} - (\gamma\boldsymbol{n} + \boldsymbol{\alpha e}) \quad (7a)$$
$$\mathcal{N} = \mathbf{N} - (\boldsymbol{\alpha n} - \boldsymbol{\gamma e}) \quad (7b)$$

However, to determine the three variation factors that will effectively transform coordinates from the entire Mid Belt in NTM system to zone 32 in UTM system precisely, the coordinates of the origin of the various systems should be used.

Data Acquisition

The orthophoto maps of Rivers State used in this study (Figure 2 and Figure 3) were acquired from MAPMART (2014) through its official web site. Data in Table 1 were obtained from ESRI (2008) and the Blue Marble Geographics (1994). The two survey plans used were obtained from Amadi (2000).

In Table 1 below, columns 2 and 3 show the NTM coordinates of the NTM Mid Belt origin and UTM Zone 32 origin. While columns 4 and 5 show their equivalent UTM coordinates. The NTM values for the Mid Belt Origin and the UTM values for the UTM Zone 32 origin were obtained from ESRI (2008). The UTM values for the Mid Belt Origin and the NTM values for the UTM Zone 32 origin were obtained from the Blue Marble Geographics (1994).

Data Presentation

The coordinates of the origin of the two systems are as shown in table 1 below:

Table 1: NTM mid Belt and UTM Zone 32 Origin Coordinates

Origin	NTM (m)		UTM (m)	
	e	**N**	**E**	**N**
Mid Belt	670553.980	0.000	444424.120	442217.205
Zone 32N	726272.964	-442267.359	500000.000	0.000

Source: ESRI (2008) and Blue Marble Geographics (1994)

Using the coordinates in table 1 in equations (4a) and (4b), the bearings (β_1 and β_2) from the Mild Belt origin to the UTM Zone 32 origin are found to be:

$\beta_1 =$ 172^0 49' 10'', and
β_2 = 172^0 50' 13''

The difference between the two bearings as in equation (3) gives the value of the deviation in bearing (**λ**) as:

λ **=** 0^0 01' 03''

Thus, substituting the values of k from equation (3) and λ in equations (5a) and (5b) for α and γ yielded

α = 0.999849953
γ = 0.000305387

Using the values of α and γ determined above and the values from Table 1, Equations (7a) and (7b) were programmed in excel to determine the parameters £ and $\mathcal{N}$ as:

£ = -226029.0858
$\mathcal{N}$ = 442422.3879.

Table 2: Comparison between Coordinates of RFD 5685 and RAB 7422 Converted by the Geographic Calculator and the Model before Applying the Correction Term

St. No	NTM E	NTM N	UTM :**Geo Cal (G)** E	UTM :**Geo Cal (G)** N	UTM: **Model (M)** E	UTM: **Model (M)** N	**ΔMG** **E**	**ΔMG** **N**
RFD 5685	505086.422	97633.341	279022.691	539962.365	279011.365	539886.832	11.326	75.533
RAB 7422	500289.935	95247.070	274223.876	537579.377	274214.869	537502.384	9.007	76.993

ΔMG = UTM Geo Cal – UTM Model

It can be observed from table 2 that the UTM coordinates obtained with the Model differs significantly from values obtained using the Geographic Calculator. For example when the coordinates of two stations were converted using the Model, the following differences from that converted by the Geographic Calculator were noted.

Investigation as to what causes the difference revealed that, as the distance of a point increases from the UTM Zone 32 origin, the value of **λ** increases. For instance, the distance between RFD 5685 and RAB 7422 from the UTM Zone 32N origin is 583452.0136m and 583086.6923m respectively. The value of **λ** between each of the stations and UTM Zone 32 origin is = 0^0 01' 17''; a difference of 14'' from **λ** used in determining the parameters. Thus there is need to introduce a correction factor in Equations (6a) and (6b).

Table 3: Values for Determining the Correction Factor (ϛ)

St. No.	**d**	**Error (*e*)** E	**Error (*e*)** N	***e*/d** E	***e*/d** N
RFD 5685	583452.0136	11.326	75.533	0.00001941130	0.0001294580
RAB 7422	583086.6923	9.007	76.993	0.00001544640	0.0001320430
Difference	365.3213057			0.00000396488	-0.000002.5853

d = Distance from UTM Zone 32N origin

From Table 3 above, correction to coordinates determined by the Model per meter for a distance of 583452.0136m is 0.00001941130m in the east and 0.0001294580m in the north.

Similarly, correction to coordinates determined by the Model per meter for a distance of 583086.6923m is 0.00001544640m in the east and 0.0001320430m in the north. However, the correction to be applied per meter supposed to be the same for any distance.

The data in last row of Table 3 shows that for a distance of 365.3213057m from the first 1m away from UTM Zone 32 origin, the correction increases by 0.00000396488m in the east and decreases by 0.000002.5853m in the north. This indicates that for every1m after the first 1m from UTM Zone 32N origin, there is an increase of $1.0853141*10^{-8}$m (0.00000396488/365.3213057) in the correction factor in easting coordinates and a decrease of $7.0767942*10^{-9}$m (0.000002.5853/365.3213057) in the northing coordinates.

This implies that at a distance of 1m from the UTM Zone 32 origin, the correction that will be applied is = 0.00001941130 – ($1.0853141*10^{-8}$*583452.0136) m = -6.3128760*10^-3 m in east and 0.0001294580 + ($7.0767942*10^{-9}$*583452.0136) m = 4.2584278*10^-3 m in north.

Thus at a distance d, the correction factor c will be

c_e = d*[(1.0853141*10^-8*d) -6.3128760*10^-3] in easting, and

c_n = d*[4.2584278*10^-3-(7.0767942*10^-9*d)] in northing.

Thus Equations (6a) and (6b) finally becomes:

$$\mathbf{E} = £+\gamma \mathbf{n} + \alpha \mathbf{e} + c_e \quad (8a)$$

$$\mathbf{N} = \mathcal{N} + \alpha \mathbf{n} - \gamma \mathbf{e} + c_n \quad (8b)$$

Equations (8a) and (8b) formed the required Simple Mathematical Model used in transforming the NTM coordinates to UTM coordinates. The values of the transformation parameters, λ, α, γ, £ and $\mathcal{N}$ obtained by equations (3), (5a), (5b), (7a) and (7b) respectively, were used to programme equations (8a) and (8b) in excel. This program was used to convert the NTM coordinates of two different surveys done by Amadi (2000) to their corresponding UTM coordinates.

Presentation of Results

Table 4 shows the results of the conversion from NTM to UTM by Geographic Calculator software. Columns 1 of Tables 5 and 6 show the NTM coordinates. Columns 2 give the UTM coordinates equivalent of columns 1 converted with the aid of the Geographic Calculator software. Columns 3 show the UTM coordinates equivalent of columns 1 converted using the Mathematical Model, while columns 4 show the difference between the results obtained by the Geographic Calculator software and that by the Mathematical Model. The UTM coordinates converted by the Mathematical Model as shown in columns 3 of Tables 5 and 6 below were overlaid on orthophoto map of Rivers State in ArcMap. The results obtained are shown in figure 2 and figure 3 respectively.

Table 4: Coordinate Conversion from NTM to UTM with the aid of Geographic Calculator Software

3	508611.636	99773.367	COORDINATE CONVERTION (NTM to UTM)		
4	INPUTE YOUR NTM COORDINATES			UTM COORDINATES	
5	EASTHINGS	NORTHINGS		EASTHINGS	NORTHINGS
6					
7	PLAN NO.: JWA/RV061/2000				
8	505086.422	97633.341		279022.688	539962.409
9	505098.206	97660.197		279034.493	539989.259
10	505126.514	97642.799		279062.791	539971.838
11	505113.823	97616.671		279050.079	539945.716
12	505101.008	97624.465		279037.269	539953.521
13	PLAN NO.: JWA/RV059/2000				
14	500268.579	95266.013		274202.591	537598.432
15	500289.071	95289.041		274223.102	537621.447
16	500311.979	95268.431		274245.997	537600.818
17	500289.935	95247.070		274223.935	537579.471

Table 5: Comparison of Coordinates of PLAN NO.: JWA/RV061/2000 Converted by the Geographic Calculator and the Model after Applying the Correction Term

St. No.	NTM		UTM :**Geo Cal**		UTM: **Model**		**Model - Geo Cal**	
	E	N	E	N	E	N	E	N
RFD 5685	505086.422	97633.341	279022.691	539962.365	279022.688	539962.409	-0.003	0.044
RFD 5684	505098.206	97660.197	279034.496	539989.215	279034.493	539989.259	-0.003	0.044
RFD 8069	505126.514	97642.799	279062.794	539971.794	279062.791	539971.838	-0.003	0.044
RFD 8070	505113.823	97616.671	279050.082	539945.673	279050.079	539945.716	-0.003	0.044
RFD 8071	505101.008	97624.465	279037.272	539953.477	279037.269	539953.521	-0.003	0.044

Table 6: Comparison of Coordinates of PLAN NO.: JWA/RV059/2000 Converted by the Geographic Calculator and the Model after Applying the Correction Term

St. No.	NTM		UTM :**Geo Cal**		UTM: **Model**		**Model - Geo Cal**	
	E	N	E	N	E	N	E	N
RAB 7421	500268.579	95266.013	274202.532	537598.338	274202.591	537598.432	0.059	0.094
RFD 3740	500289.071	95289.041	274223.043	537621.354	274223.102	537621.447	0.059	0.093
RFD 3739	500311.979	95268.431	274245.939	537600.724	274245.997	537600.818	0.058	0.094
RAB 7422	500289.935	95247.070	274223.876	537579.377	274223.935	537579.471	0.059	0.094

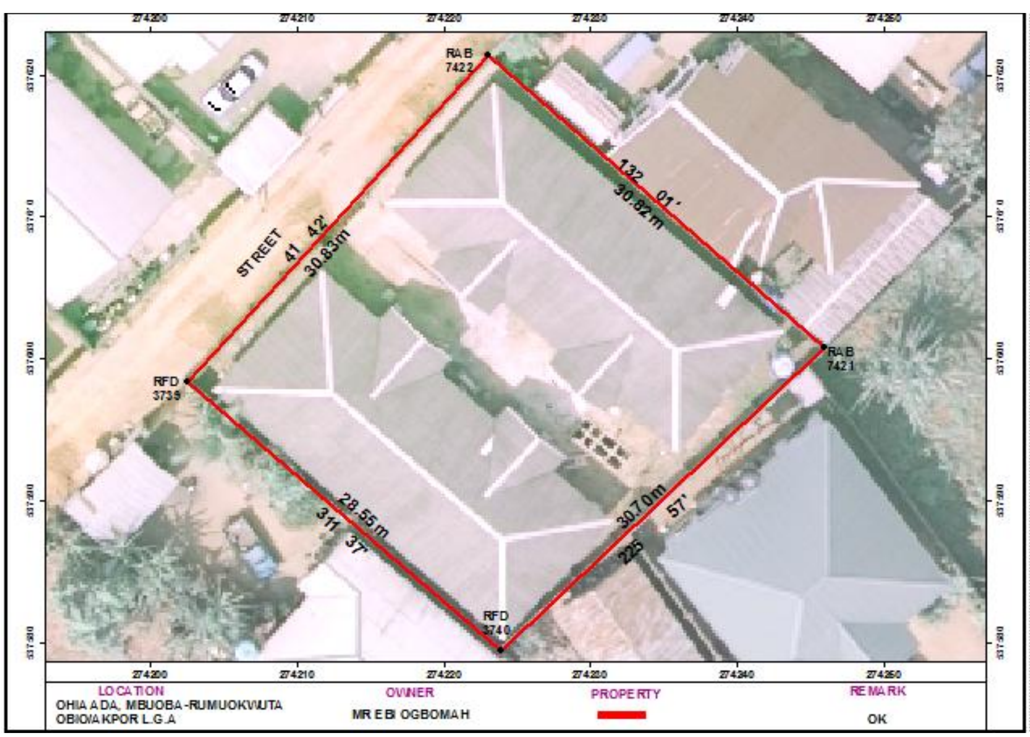

Fig 2: Plan No.: JWA/RV059/2000 Overlaid on the 2005 Orthophoto Map of Rivers State

Fig 3: Plan No.: JWA/RV061/2000 Overlaid on the 2005 Orthophoto Map of Rivers State

Analysis of the Results

The results show that the UTM coordinates obtained by the Simple Mathematical Model as compared with the one obtained by the Multiple Regression Equations as programmed in the Geographic Calculator software gave an average difference of -0.003m, 0.044m in easting and 0.059m, 0.094m in northing. Investigation show that if more numbly of data points are used, the accuracy reduces randomly and significantly. The error seems to be a factor of the distance of each point from the UTM Zone 32 origin. This is because the Model uses only two points (parameters defining the NTM Mid Belt origin and that defining the UTM Zone 32 origin) to determine the transformation parameters (**λ, α**, **γ,** £ and $\mathcal{N}$). In this paper the averages of the parameters £ and $\mathcal{N}$ determined from the two points were used. Since the error does not follow a progressive pattern, it is difficult to apply a correction that will transform coordinates precisely. This indicates that the Model is not good enough for geodetic or engineering purposes. However, the results show that the Model is significant for plane surveys where calculating land composition and topography considers a set expanse of land as a flat plane. Figures 2 and 3 further confirmed this.

Conclusions

Conversion of NTM coordinates to UTM coordinates using a Simple Mathematical Model has been discussed. From the foregoing analysis, a Model that converts coordinates with an error of 0.044m in easting and 0.094m in northing is good enough for small scale cadastral surveys such as revalidation of plans, layout revision and street map update etc. Although it may not be good enough for precise engineering purposes, the overall result from the model however indicates that the objective of this research has been achieved. It is therefore concluded that the conversion of coordinates from NTM to UTM using this Model is a promising alternative to the laborious, rigorous and difficult approach.

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