Modeling Wind Effect on Waste Stabilization Pond Performance

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Abstract

Wind has an important effect on the behavior of ponds as it induces vertical mixing of the pond contents. Good mixing ensures a more uniform distribution and higher dispersion number within the pond and hence a better degree of waste stabilization. This study developed a mathematical model for the prediction of Coliform bacteria ($^{ce}/_{co}$) on waste stabilization pond performance. The model was developed based on two-dimensional steady dispersed flow model and Hulbult's (1944) boundary conditions. The solution of the equation was restricted to method of separation of variable and fourier series expansion. The model on wind effect was carried out with different wind speed 2.27m/s, 1.88m/s and 1.64m/s respectively directed at different tanks at inlet, outlet and the side of the tanks labeled, B, C and D, while tank A was under control condition. The model was verified using the laboratory scale model ponds (LSWSP) experimental results. The solution of the model was obtained by writing Fortran computer programme for the computation of coliform bacteria ratio ($^{ce}/_{co}$). The coefficient of correlation between the measured and the predicted values ranges between 0.8710 to 0.9980 indicating that the prediction are very good.

Keyword: Dispersion number, waste stabilization pond, coliform bacteria, fourier series, separation of variable.

Introduction

The primary purpose of wastewater treatment is the reduction of pathogenic contamination, coliform bacteria, suspended solids, oxygen demand and nutrient enrichment. Those treating raw wastewater are referred to as facultative ponds, lagoons or oxidation ponds. Their purpose is to further reduce suspended solids, BOD, faecal micro-organisms and ammonia in the plant effluent. Wastewater stabilization ponds (WSPs) are a cheap and effective way to treat wastewater in situations where the cost of land is not a factor. Not only has it been found to be one thousand times better in destroying pathogenic bacteria and intestinal parasites than the conventional treatment plants (Mara and others, 1983). It is also more economical (Arthur, 1983). It is simple to construct, operate and maintain and it does not require any input of external energy. Although a waste stabilization pond system usually requires large land area because of its long detention time which is attributable to its complete dependence on natural treatment process, it will still be very suitable in several African countries and communities where land acquisition is not a problem. Besides, its efficiency depends on the availability of sunlight and high ambient temperature which are the prevailing climatic conditions in most of these communities.

Many characteristics make waste stabilization pond substantially distinguished from other wastewater treatment methods. This includes design construction and operation simplicity, cost effectiveness, low maintenance requirements, easily adoptive for upgrading and high efficiency.

Conventional treatment of liquid wastes involves mechanical treatment systems, and is the norm in developed countries. However, they are not the best option for less developed countries. Indeed, conventional treatment schemes were developed due to climatic and area constraints. These constraints are often not the case in developing countries. Moreover, the use of energy intensive mechanisms is not desirable in less developed countries, where energy supply is not reliable. Further, conventional treatment facilities require regular high-skilled maintenance, a thing that is either too expensive or impossible to find in developing countries. Wind has an important effect on the behaviour of facultative ponds as it induces vertical mixing of the pond liquid. Good mixing ensures a more uniform distribution of Biochemical Oxygen Demand (BOD), dissolved oxygen (DO), Coliform bacteria and algae and, hence, a better degree of waste stabilization.

In the absence of wind-induced mixing, the algal population tends to stratify in a narrow band some 20cm thick during day light hours (Mara and others, 2001). This concentrated band of algae moves up and down through the top 50cm of the pond in response to changes in incident light intensity and causes large fluctuations in effluent quality (especially BOD and suspended solids). Hence wind action promotes mixing and reaeration within the pond system and operation. Wind sweeping over water surfaces creates a zone of circulation within the top surface of the pond called the epiliomnion. The wind-created velocity is used up in transporting the water through the length of the fetch and back to its point of origin in the return flow (Wright, 1972; and Gallagher and others, 1973). There has been growing activity in the study of wind generated circulation in lakes and Lagoons. Representative studies in this direction are those of (Gallagher and others, 1973 and Gedney and others, 1972) respectively. Other investigators have studies circulation patterns in lakes and Lagoons. The effects of wind on velocity distribution and wave generation have been studied by (Wu, 1973). In many cases of practical importance, the need to know the velocity distribution in a lake or Lagoon stems from the need to know the distribution of transportable substances, such as the BOD and DO. In these cases, wind action has more than a single role to play, not only does it determine the main features of the velocity distribution but also it establishes the magnitude of the turbulent diffusion and surface reaeration.

The longitudinal dispersion coefficient in a pond or wide channel can be calculated under various combinations of stream flow and wind conditions. The effect of wind, which produces drift currents in the stream, on dispersion has hitherto been neglected. The mechanism of longitudinal dispersion in turbulent shear flow was discussed first by (Elder, 1959). Restricting his study to a long straight circular pipe, Taylor not only proposed a scheme of calculation, but also verified his analytic results experimentally. Other extensive experiments have also been conducted in laboratory Flumes by (Elder, 1959, Fischer, 1966 and 1967) respectively and in natural streams by (Thackstone and Krenkel, 1967).

Also (Wu, 1969) expressed the combined velocity distribution in a channel as:

$$U = U_m \left(\frac{y}{d} \right)^{\frac{1}{7}} + V_s \left[1 - \left(1 - \frac{y}{d} \right)^{\frac{1}{7}} \right] \qquad - \qquad - \qquad - \qquad - \qquad - \qquad - \qquad - \qquad (1)$$

in which U_m is the maximum channel velocity under no wind, V_s is the surface drift current, and y is the distance from the bottom of the channel while d is the depth of the channel. Several researchers such as (Sweeney and others 2000; Benque and others, 1982) have demonstrated the importance of solar radiation and wind speed in determining the range of bulk values and stratification of each of the pond parameters throughout the pond which include such biological activity indicators such as temperature, DO and pH to changes in environmental conditions. The wind effect on the top surface of the model was simulated by applying a shear stress τ (N/m^2) consistent with the relationship developed by (Benque, Hauguel and Viollet 1982) as:

$$\tau = C_d \ell_o U_{10}^2 \qquad - \qquad - \qquad - \qquad - \qquad - \qquad - \qquad - \qquad - \qquad - \qquad (2)$$

where C_d is the drag coefficient and ranges from 0.9 x 10^{-3} for $U_{10} > 10$m/s, U_{10} is the magnitude of wind velocity at height 10m (m/s), ℓ_o is the density of air.

(Brissaud, and Others, 2000) stated that particular attention has to be paid to shortest retention time, because it plays a key role in micro-organism removal performance. feacal coliform removal in WSPs is highly dependent on shortest water retention time. Water retention time is, together with solar radiation and temperature one of the most important factors which influence pathogenic micro-organism removal in stabilization ponds. This is the reason why a number of tracer tests have been performed during the eighties and in recent years by several researchers such as (Racault and Douat, 1984; Murescos do Monte and Mara, 1987; Nameche and Vasel; 1998; Brissaud and Others, 2000). Many results confirm that the old Marais's assumption which states that ponds behave as perfectly mixed reactors is fairly valid for medium and long term water transfer. Brissaud further stated that in sunny and low wind periods a clear stratification are observed during the day with high temperature, DO, pH and red OX potential in the epilimnion. Temperature varied rapidly at the surface with maximum differences of more than 15^{0}C between night and day. Deeper in the pond, the range of the temperature variation diminished.

During day time, temperature decrease from the surface to the bottom together with DO, pH and red – OX potential but, the stratification vanishes gradually during the night.

Fig. 1: Vertical Profile of lwsp for the study of Wind effect
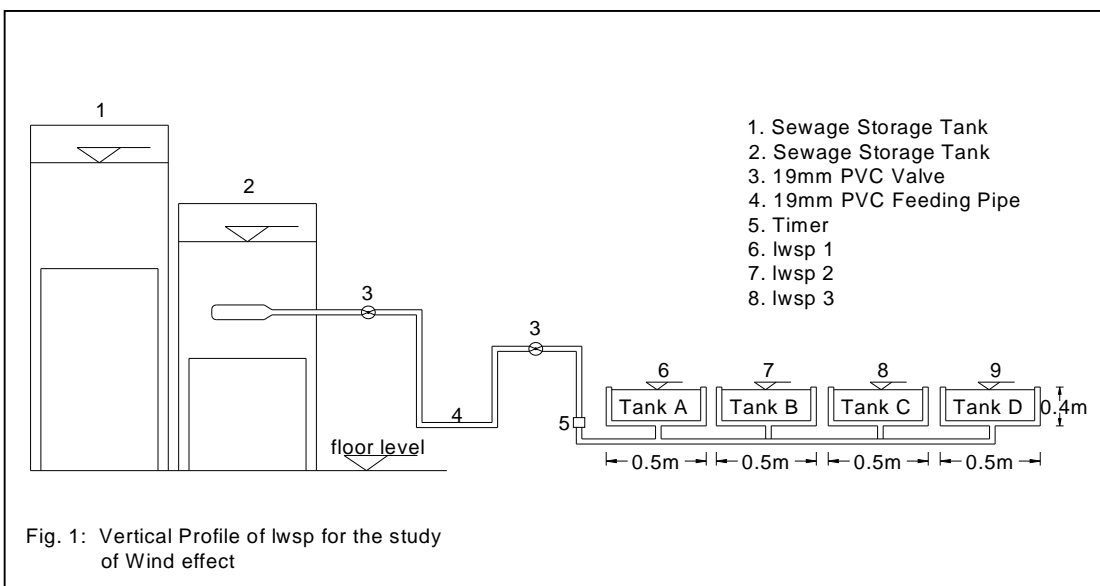
Mixing appears to be the main feature of pond dynamics. The mechanisms by which it is driven are daily variations of water temperature and density fields and wind friction at the surface of the pond. Both mechanisms are directly ruled by climatic conditions. Other researchers like (XU; Brissaud and Fazio; 2004) stated that a better knowledge of fluid dynamics may help to understand and predict better bacterial die-off kinetics and improve the quality of disinfection in the pond.

Material and Methods

Laboratory Experimental Set Up

The experimental set up consisted of four rectangular units, made of thick metal sheets to study the influence of wind effect (wind speed and direction) and mixing in WSP performance as shown in Figure 1. In each case, one of the rectangular units of the laboratory scale waste stabilization pond (LSWSP) was operated under control condition while the three others were operated under different wind speeds of (2.27m/s, 1.88m/s and 1.64m/s) and also under different directions such as – inlet, outlet and side directions respectively. The LSWSP inlets were connected to a flow inducers to obtain a constant influent flow. Feedlines of 19mm diameter (PVC) pipes with 19mm diameter gate values to regulate the influent flow were connected from the Tanks to the 500L polythene vessel capacity feed Tank with a tee joint to enhance even distribution within all the Tanks.

Two 500L polythene vessels with a stirrer will be used as the feed Tank to which feed lines were connected to facilitate continuous operation of the system. The feed Tank were placed at an elevation of 2m and 1.5m respectively with the Tanks as shown in Figure 1 to enable the wastewater enter the Tanks through gravity and also to allow the influent drop freely into all the Tanks to facilitate dispersion within the Tanks. The effluent discharge through a 19mm diameter PVC pipes separated with 19mm diameter gate values to minimize back flows. The experiments were conducted inside the sanitary laboratory in Civil Engineering Department, University of Nigeria, Nsukka under normal room temperature and the Tanks illumination were accomplished by providing a set of fluorescent bulbs fitted to a wooden stand. The system were set up for a few weeks to allow the system attain steady state conditions.

Laboratory investigation of wind effect on WSP performance was conducted with wind speed operated at 2.27m/s, 1.88m/s and 1.64m/s speeds for Tanks B, C and D while Tank A was under control. Wastewater samples were collected from the first of a two facultative pond system in series, measuring 120m by 30m by 0.2m respectively. It has a thick sediment layer and empties into the second pond of approximately the same size but almost covered with vegetation.

The pond system is used for further treatment of domestic wastewater from the Nsukka campus of the University of Nigeria, Nsukka (UNN). The pond system receives effluent after screening and digestion and sedimentation in an imhoff Tank and serves a population of about 30,000.All analyses were undertaken according to the methods described in the standard methods for the examination of water and wastewater, APHA (1998). The physicochemical parameters observed were temperature, dissolved oxygen, pH, chemical oxygen demand (COD), suspended solids (SS), and Coliform bacteria. Elaborate tracer studies involving the collection and analysis of 500 samples were performed in order to determine the hydraulic efficiency of each Tank and the dispersion number (d) were obtained using Levenspiel and Smith method, Levenspiel, (1957).The two dimensional steady dispersion equation was solved by the method of separation of variable. Computer programme based on Fortran Language was used to obtain values from the wind effect model for ($^{ce}/_{co}$).

Development of Mathematical Model for Wind Effect in Waste Stabilization Ponds Performance

The two dimensional unsteady dispersion model equation with first order reaction can be written as:

$$\frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} - U \frac{\partial c}{\partial x} - V \frac{\partial c}{\partial Y} + KC \qquad - \qquad - \qquad - \qquad - \qquad - \qquad (3)$$

where:

c = the concentration of bacteria number, per 100ml;

u = the function of both the wind and pond velocities; m per day

k = the bacterial die-off rate coefficient per day;

y = the pond depth varying from O at the surface to H at the bottom in m;

x = the pond length varying from O at the inlet to L at the pond outlet in m;

v = the pond settling velocity in meter per day.

t = time in days

D_y = the vertical dispersion coefficient in the pond, in m^2 per day.

D_x = the longitudinal dispersion coefficient in the pond in m^2 per day

The combined velocity distribution in the pond can be expressed according to W$_u$ (1969) as:

$$U \quad = \quad U_m \left(\frac{y}{H}\right)^{\frac{1}{7}} + Vs \left[1 - \left(1 - \frac{y}{H}\right)^{\frac{1}{7}}\right] \qquad - \qquad - \qquad - \qquad - \qquad - \qquad (4)$$

where

U_m = the maximum pond velocity under no wind, meter per day;

V_s = the surface drift current caused by wind effect, in meter per day;

y = the distance from the pond bottom in meter, and

H = the depth of the pond in meter

Assuming that the wind occur only within a short interval of time and under steady state conditions equation (3) can be written to account for bacterial die-off rate coefficient (k), dispersion coefficient (D) and the combined velocity (U) as:

$$D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} = U \frac{\partial c}{\partial x} - V \frac{\partial c}{\partial y} + KC = O \quad - \qquad - \qquad - \qquad - \qquad - \qquad - \qquad (5)$$

The solution to equation (5) under steady state condition can be obtained with the following boundary conditions. Using Hulbult's boundary conditions, Hulbult (1944) for the x – axis, it implies that:

$$C_{(0-)} = C_{(0+)} = C_O, \ x = O \qquad \text{(6)}$$

$$\frac{\partial c}{\partial x} = O, \qquad x = L \qquad \text{(7)}$$

Also for the vertical axis, using the following boundary conditions, the solution to equation (5) under steady state condition can be obtained.

At the pond surface layers,

$$\frac{\partial c}{\partial y} = O \quad at \quad y = O \qquad \text{(8)}$$

and at the pond bottom layer,

$$\frac{\partial c}{\partial y} = O \quad at \quad y = H \qquad \text{(9)}$$

where $C_{(0-)}$ is the Coliform number in the pipe, just before the wastewater enters the pond. $C_{(0+)}$ is the number of coliform within the pond entrance. Equation (7) expresses the boundary condition at the pond outlet where $x = L$, and C_O is the influent bacteria number.

Solving equation (5) by method of separation of variable, it becomes (see the Appendix).

let

$$x = A_1 e^{m_1 x} + B_1 e^{m_2 x} \qquad \text{(18)}$$

and

$$y = A_2 \, Cosn_1 y + B_2 \, Sinn_2 y \qquad \text{(19)}$$

where:

$$m_1 \quad = \quad \frac{u + (u^2 - 4D_x \lambda^2)^{1/2}}{2D_x} \qquad \text{(19)}$$

$$m_1 \quad = \quad \frac{u - (u^2 - 4D_x \lambda^2)^{1/2}}{2D_x} \qquad \text{(21)}$$

$$n_1 \quad = \quad \frac{V + \left[V^2 - 4D_y\left(k - \lambda^2\right)\right]^{1/2}}{2D_y} \qquad \text{(22)}$$

and

$$n_2 \quad = \quad \frac{V - \left[V^2 - 4D_y\left(k - \lambda^2\right)\right]^{1/2}}{2D_y} \qquad \text{(23)}$$

otherwise,

$$C(x_1 \, y) = \sum_{n=1}^{\infty} D_n T_n \qquad \text{(37)}$$

and

$$C(x_1 \, y) = \sum_{n=1}^{\infty} D_n Cos \frac{\pi n}{H} y \left[e^{m_1 x} - \frac{m_1 e^{m_1 L}}{m_2 e^{m_2 L}} e^{m_2 x} \right] \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad (38)$$

From equation (4) and (37)

$$\sum_{n=1}^{\infty} D_n Cos \frac{\pi n}{H} y \left[1 - \frac{m_1 e^{m_1 L}}{m_2 e^{m_2 L}} \right] = C_o \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad (39)$$

By fouries series expansion,, D_n is obtained as:

$$D_n = \frac{C_o \dfrac{H}{\pi n} Sin \dfrac{\pi n H}{H}}{\left[\dfrac{U - 2D_x^{1/2}\lambda}{D_x} - \dfrac{U - 2D_x^{1/2}\lambda}{D_x^{1/2}\lambda} \ell \dfrac{U - 2D_x^{1/2}\lambda}{D_x} \right] x \left[\dfrac{H}{2} \right] - \left(\dfrac{H}{2\pi n} \right)^2 Sin \dfrac{\pi n H}{H}} \qquad \text{-} \qquad \text{-} \qquad (40)$$

Substituting D_n into equation (37) and after mathematical induction.

$$\frac{C_e}{C_o} = \frac{\dfrac{H}{\pi n} Sin \dfrac{\pi n H}{H} Cos \dfrac{\pi n y}{H} \left[\ell \left(\dfrac{U - D_x^{1/2}\lambda}{D_x} \right) L \right] \sum_{n=1}^{\infty} - \left(\dfrac{U - D_x^{1/2}\lambda}{D_x^{1/2}\lambda} \right) \ell \dfrac{U - 2D_x^{1/2}\lambda + D_x^{1/2}\lambda L}{D_x}}{\left[\left(\dfrac{U - D_x^{1/2}\lambda}{D_x} \right) - \left(\dfrac{U - D_x^{1/2}\lambda}{D_x^{1/2}\lambda} \right) \ell \sum_{n=1}^{\infty} - \left(\dfrac{U - D_x^{1/2}\lambda}{D_x^{1/2}\lambda} \right) \ell \dfrac{U - 2D_x^{1/2}\lambda}{D_x} \right]}$$

$$\left[x \frac{H}{2} - \left(\frac{H}{\pi n} \right)^2 Sin \pi n \right] \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad \text{-} \qquad 41$$

where:

$$D_y = \alpha h U_*$$

Result and Discussion

The effects of wind speed and direction were studied with the LWSP using standing fan as a source of wind generation under different speed at high, medium and low speed when converted results in the values of 2.27m/s,1.88m/s and 1.64m/s representing high, medium and low wind speed. The wind (fan) was controlled at different speed for Tanks B, C, D and Tank A was under control condition. Analysis of the samples collection with a water column sampler at 0.1m depths showed some homogeneity (uniform condition) for the coliform reduction number (Ce/Co) at high medium and low speed. The coliform reduction number changes at the pond bottom (depth) under different wind speed as shown in figures 2,3, and 4 respectively, it was observed that at Tanks B, the coliform reduction number at the pond bottom was within 0.15 to 0.16, while at medium speed (1.88m/s), the coliform reduction number started increasing from 0.20 to 0.27 and at low speed (1.64m/s), the coliform reduction number further increased from 0.35 to 0.37. In the absence of wind under the control condition for Tank A, the coliform reduction number at the bottom of the pond was at its maximum value of 0.40 and 0.41.

Therefore, it can be concluded that wind sweeping over wastewater pond surfaces has an important effect on the behavior of its performance as it reduces the coliform number. This implies that the efficiency of the pond will increase and the odour problem will also be minimized.

Verification and Response of Wind Effect Model

The data of the laboratory scale waste stabilization pond developed in this study were used to evaluate the response of wind effect model of equation (41) developed in this study. Comparisons of the measured experimental effluent coliform bacteria data and those predicted from the model were made to show the responses accuracy and sensitivity.

The plot of predicted values $\left(\dfrac{Ce}{Co}\right)_P$ and measured values $\left(\dfrac{Ce}{Co}\right)_m$ versus depth for the laboratory scale waste stabilization ponds are as presented in Figures 2, 3, 4, 5, 6 and 7, respectively.

Comparing the laboratory experimental data with those predicted by the proposed model gave the coefficient of correlation which ranges from 0.8710 to 0.9980 indicating that the model of wind effect developed in this study performed with a high degree of accuracy in the prediction of the coliform reduction ratio.

The model application in design, evaluation and performance is through the computer aided programme based on Fortran Language where the coliform reduction ratio $\left(\dfrac{Ce}{Co}\right)$ was obtained.

Although wind is a random phenomenon that can be imposed at any time, the application of wind effect model should be taken up as a future research work.
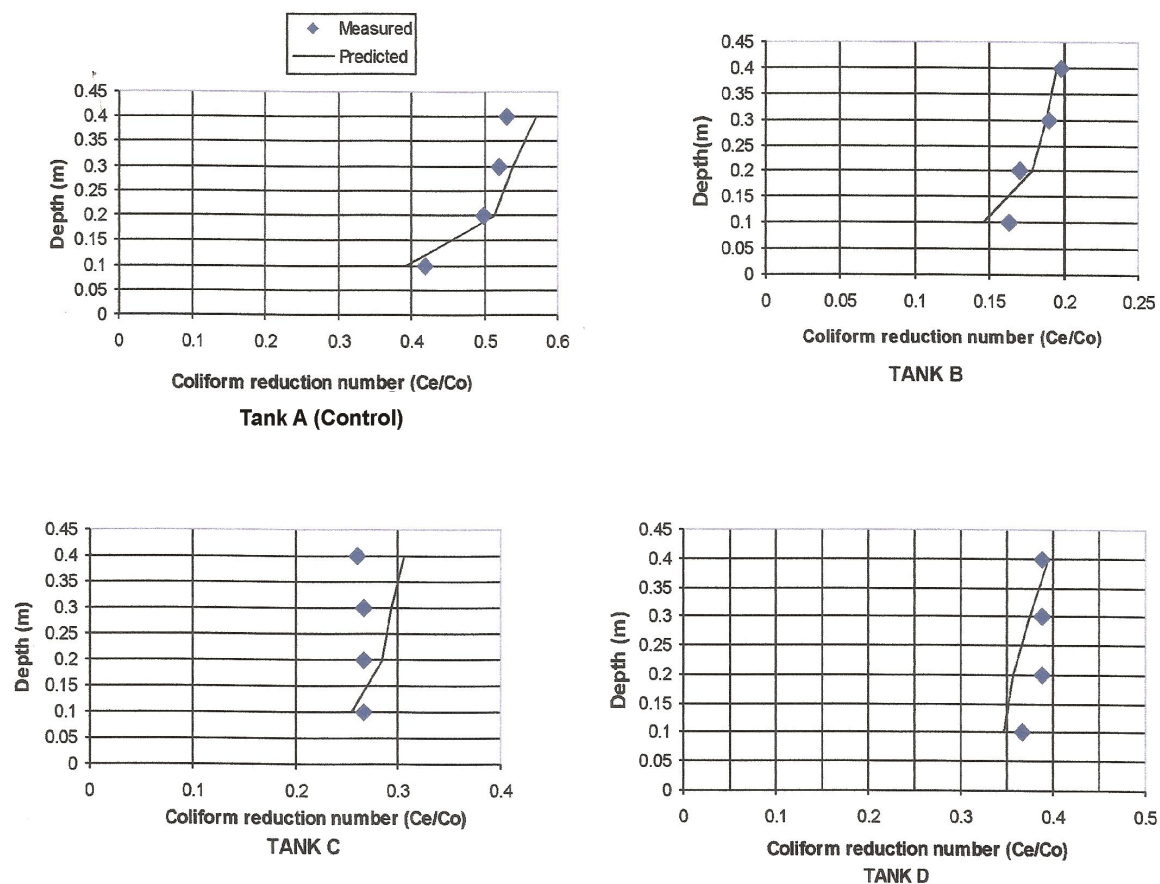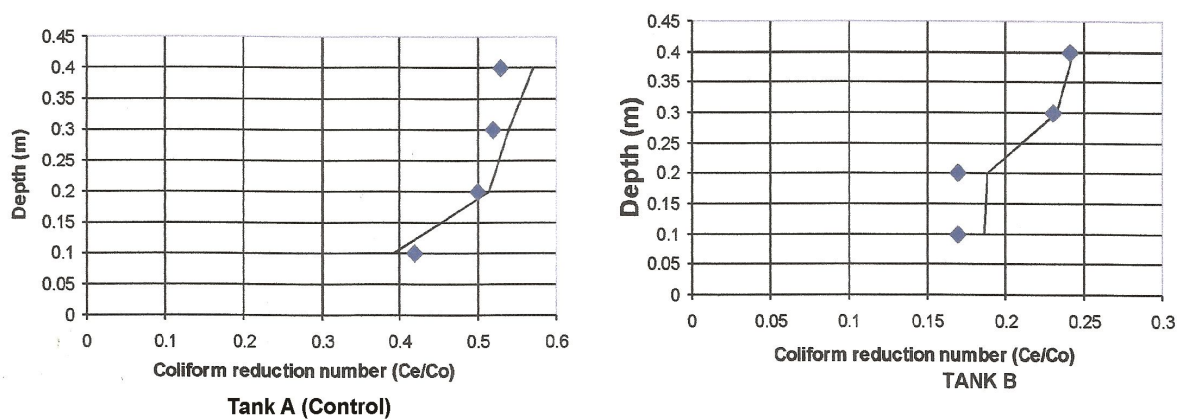
Figure 2 : Measured and Predicted Coliform reduction number (Ce/Co)
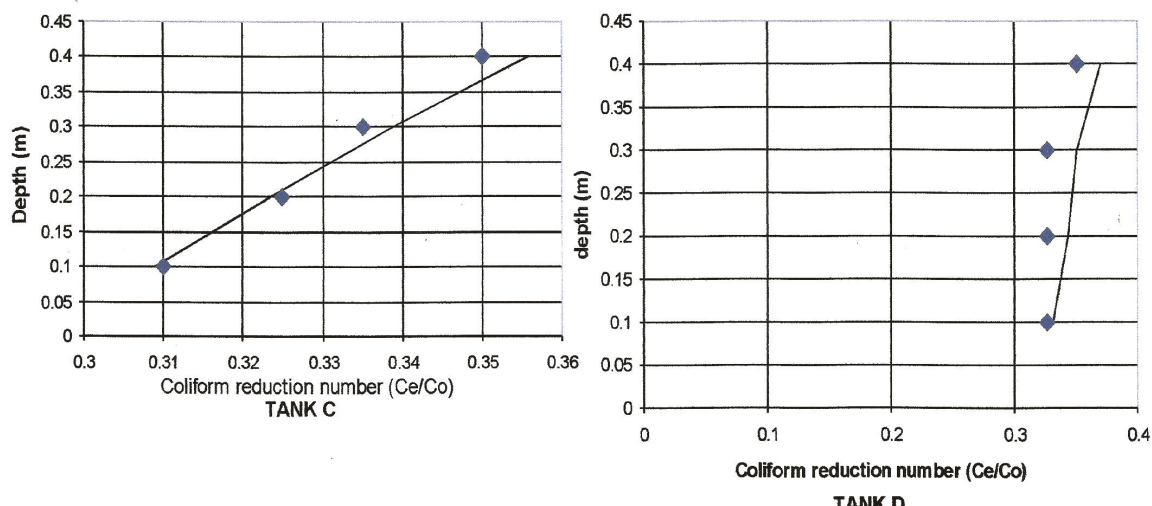due to different wind speed for Tank A, B,C and D - Inlet direction

Figure 3 : Measured and Predicted Coliform reduction number (Ce/Co) due to different wind speed for Tank A, B,C and D - Outlet direction
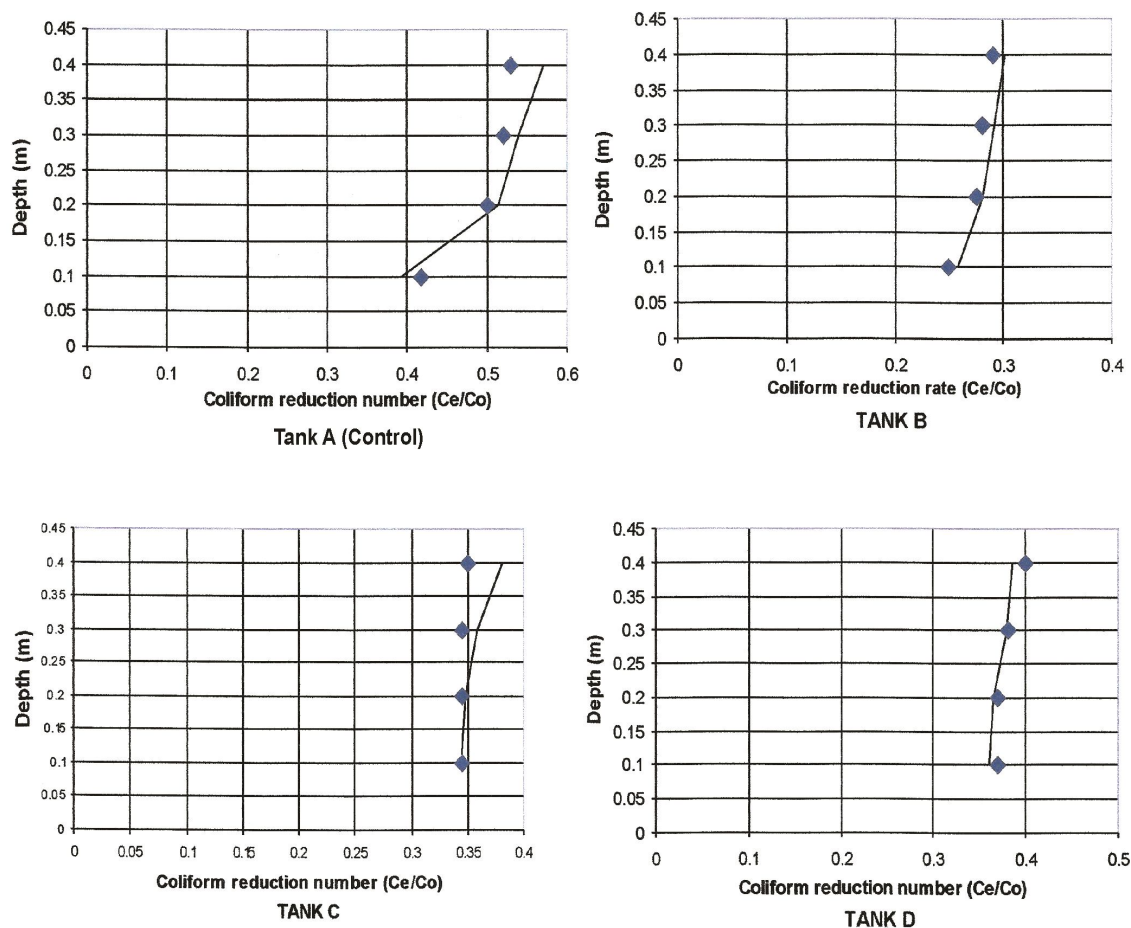

Figure 4 : Measured and Predicted Coliform reduction number (Ce/Co) due to different wind speed for Tank A, B,C and D -Side direction
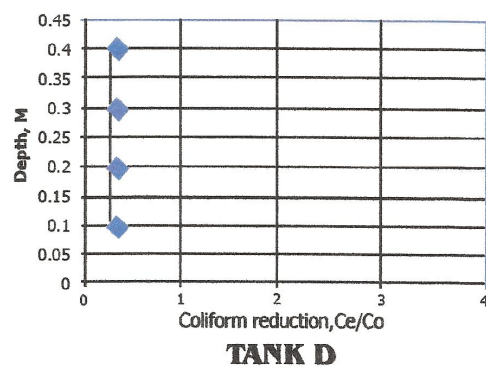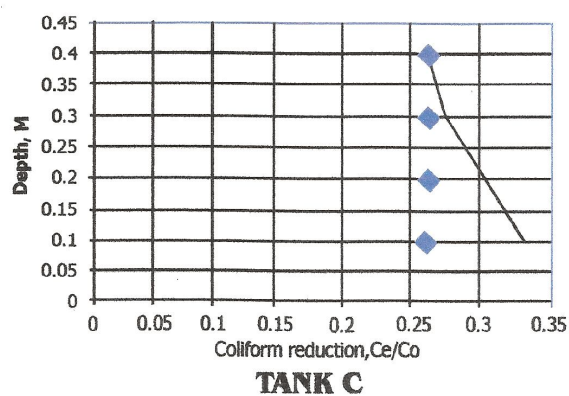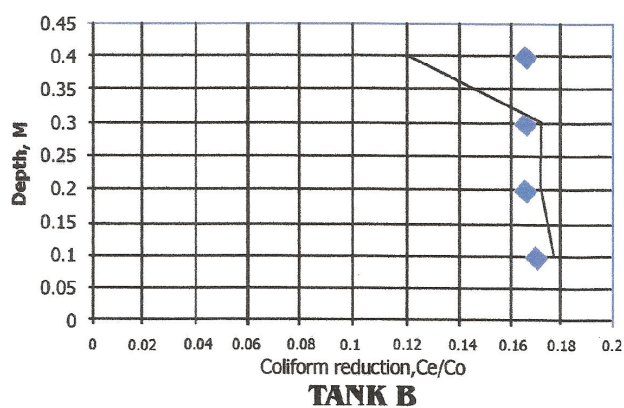
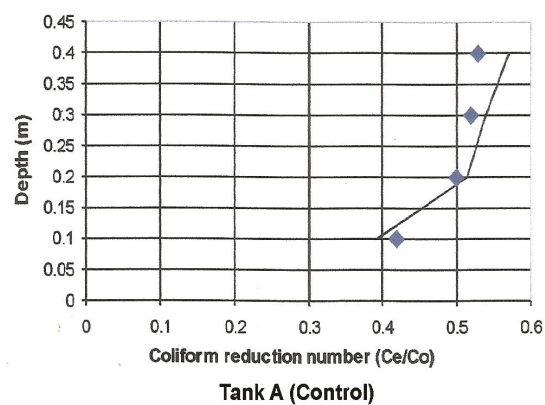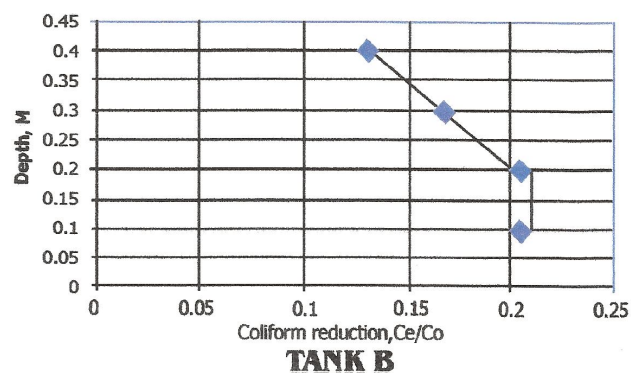**Tank A (Control)**

TANK B

TANK C

TANK D

Figure 5 Measured and Predicted Coliform reduction number (Ce/Co) due to different wind speed for Tank A, B, C and D - Inlet direction
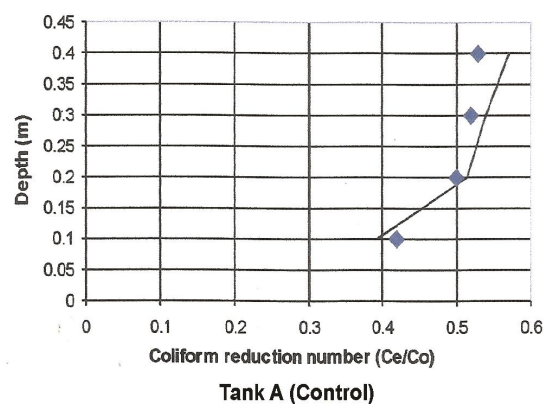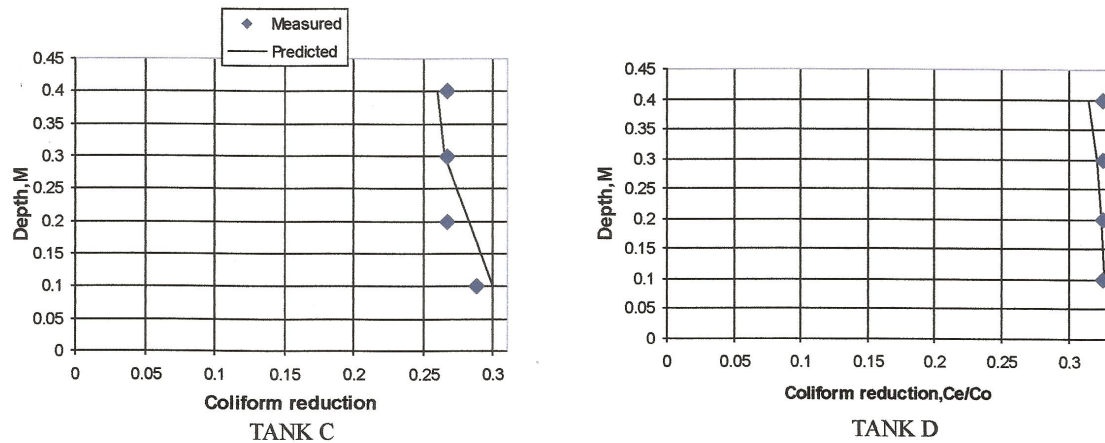
Tank A (Control)

TANK B

Figure 6 : Measured and Predicted Coliform reduction number (Ce/Co) due to different wind speed for Tank A, B, C and D - Outlet direction
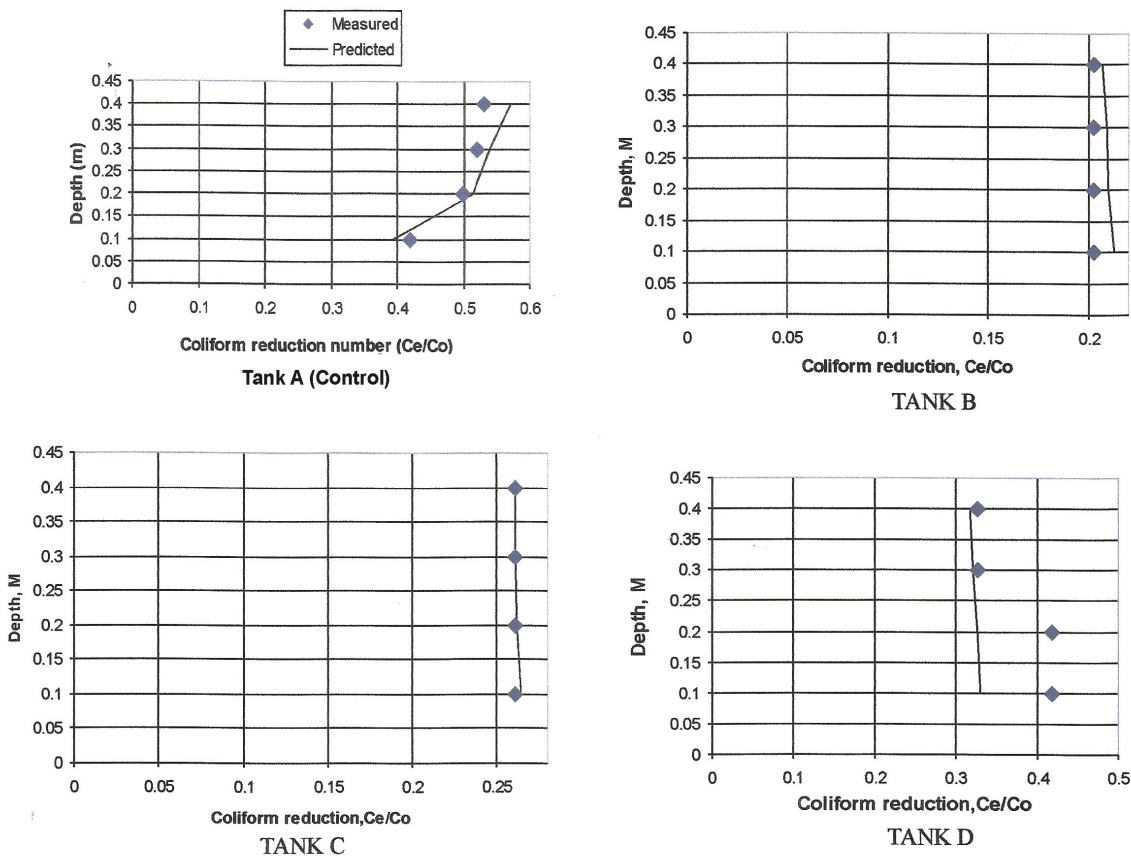

Figure 7 : Measured and Predicted Coliform reduction number (Ce/Co) due to different wind speed for Tank A, B, C and D - Side direction

Conclusion

With data from the laboratory, it has been demonstrated that the model of wind effect and mixing predicts more accurate values when compared with the measured values. Hence it is recommended for pond design, evaluation and performance. By sampling and determining the coliform reduction number (Ce/Co), it has been shown that, the coliform reduction number reduces with wind speed while it increases under no wind condition. Again, it was observed that homogeneous condition exists in ponds due to the influence of wind action.

Therefore, the proposed mathematical model for the determination of wind effect in WSPs performance provides satisfactory results with coefficient of correlation, from measured to predicted values in the range of 0.8710 and 0.9980.

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Appendix

$$D_x X^{11} Y + D_y X Y^{11} - UX^1 Y - VXY^1 + KXY = 0 \quad - \quad - \quad - \quad 10$$

and

$$\frac{D_x X^{11} - UX^1}{X} = \frac{VY^1 - D_y Y^{11} - KY}{Y} = -\lambda^2 \quad - \quad 11$$

Where λ *can be any number.*

Therefore:

$$\frac{Dx\, X^{11} - UX^1}{X} = -\lambda^2 \quad - \quad - \quad - \quad - \quad - \quad - \quad 12$$

and

$$\frac{VY^1 - D_y Y^{11} - KY}{Y} = -\lambda^2 \quad - \quad - \quad - \quad - \quad 13$$

Equation (12) and (13) can be expressed as:

$$D_x X^{11} - UX^1 + \lambda^2 x = 0 \quad - \quad - \quad - \quad - \quad - \quad - \quad 14$$

and

$$D_y Y^{11} - VY^1 + (K - \lambda^2)\, Y = 0 \quad - \quad - \quad - \quad - \quad - \quad - \quad 15$$

where the characteristic equation can be written as:

$$D_x m^2 - U_m + \lambda^2 = 0 \quad - \quad - \quad - \quad - \quad - \quad - \quad 16$$

and

$$D_y n^2 - V_n + (K - \lambda^2) = 0 \quad - \quad - \quad - \quad - \quad - \quad - \quad 17$$

where the constant A_1, B_1, A_2, and B_2 can be obtained from the various boundary conditions respectively. From equation (8) and (19)

$$n_1 A^2 Sin\, O - B_2 n_2 Cos\, O = 0 \quad - \quad - \quad - \quad - \quad - \quad 24$$

Therefore, $B_2 = 0$

Similarly from equation (9) and (19);

$$n_1 A_2 Sin\, n_1 H - B_2 n_2 Cos\, n_2 H = 0 \quad - \quad - \quad - \quad - \quad 25$$

Thus,

$$A_2 n_1 Sin\, n_1 H = 0 \quad - \quad - \quad - \quad - \quad - \quad - \quad 26$$

but $A_2\, n \neq 0$

Therefore:

Sin $n_1 H = 0$

$$n_1 H = Sin^{-1}(0) = \pi n \quad - \quad - \quad - \quad - \quad - \quad - \quad 27$$

and

$$n_1 = \frac{\pi n}{H} \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad 28$$

Hence,

$$\frac{V + \left[V^2 - 4D_y\left(K - \lambda^2\right)\right]^{\frac{1}{2}}}{2D_y} = \frac{\pi n}{H} \quad - \quad - \quad - \quad - \quad - \quad - \quad 29$$

Simplifying equation (29), gives

$$\lambda = \left(\frac{D_y \pi^2 n^2}{H^2} - \frac{V\pi n}{H} + K \right)^{1/2} \qquad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad 30$$

and

$$y = A_2 \, Cos \, \frac{\pi n}{H} y \qquad - \quad - \quad - \quad - \quad - \quad - \qquad 31$$

Also from equation (5) and (17)

$$A_1 m_1 e^{m_1 L} + B_1 m_2 e^{m_2 L} = 0 \qquad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad 32$$

where

$$B_1 = -\frac{A_1 m_1 e^{m_1 L}}{m_2 e^{m_2 L}} \qquad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad 33$$

and

$$X = A_1 e^{m_1 x} - \frac{A_1 m_1 e^{m_1 L}}{m_2 e^{m_2 L}} e^{m_2 x}$$

which gives

$$X = A_1 \left(e^{m_1 x} - \frac{m_1 e^{m_1 L}}{m_2 e^{m_2 L}} e^{m_2 x} \right. \qquad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad 34$$

Therefore by principle of superposition using equation (30) and (33) respectively,

$$C(X_1 Y) = D \, Cos \, \frac{\pi n}{H} y \left[e^{m_1 x} - \frac{m_1 e^{m_1 l}}{m_2 e^{m_2 L}} e^{m_2 x} \right] \qquad - \quad - \quad - \quad - \quad - \qquad 35$$

Equation (34) satisfies equations (5), (6), (7) but does not satisfy equation (4).

Therefore the general solution to equation (3) can be obtained as:

$$C (X, Y) = D_n T_n \qquad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad 36$$

let assume that C (X, Y) can be developed in an infinite series, such that

$$C (X_1 Y) = D_1 T_1 + D_2 T_2 + D_3 T_3 + \qquad - \quad - \quad - \quad - \quad - \quad - \qquad 37$$