Power Flow Analysis of the Nigerian Transmission System Incorporating Facts Controllers

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Abstract

This paper presents the results of power flow analysis of Nigerian power system incorporating Flexible AC Transmission Systems (FACTS) controllers such as Static Synchronous Compensator (STATCOM), High Voltage Direct Current- Voltage Sourced Converter (HVDC-VSC) and Unified Power Flow Controller (UPFC) for voltage magnitude control, active and reactive power flow control. The FACTS controllers power injection models with the voltage expressed in rectangular form were presented in this work. The steady-state models of the FACTS controllers produced algebraic equations which were combined with power system network algebraic equations and these were solved using Newton-Raphson solution technique. A MATLAB based program, Flexible Alternating Current Transmission System Power Flow (FACTSPF) for steady-state analysis of power was developed. Power flow analysis of the Nigerian transmission system conducted confirmed results of earlier works that the system is weak with voltage limit violations and high power losses. Application of the FACTS controllers resulted in the voltage profile of the system being brought within acceptable limit of $\pm$ *10%. In addition they also assisted in reducing in high power losses of system depending on type and number of FACTS controller installed.*

Keywords: *FACTS, Newton-Raphson, STATCOM, HVDC-VSC and UPFC*

1. Introduction

In interconnected power systems, such as Nigerian transmission power system, it has become important to fully utilise the existing transmission facilities instead of building new power plants and transmission lines that are costly to implement and involve long construction times[1]. Flexible Alternating Current Transmission Systems (FACTS) controllers have been introduced in power systems to solve the above problems. FACTS make it possible to control the voltage magnitude of a bus, active and reactive power flows through transmission line of a power system. There are different types of FACTS controllers configurations, but those based on voltage sourced converter (VSC) concept have several attractive features such as faster control responses, lower output distortion and being able to improve dispatch flexibility by circulating active power between their AC and DC terminals[2]. Amongst the widely used VSC based FACTS controllers are Static Synchronous Compensator (STATCOM), High Voltage Direct Current – Voltage Sourced Converter (HVDC-VSC) and Unified Power Flow Controller (UPFC).

As the Nigerian Government is finalising the eventual deregulation of her power system it is important that the investing companies be avail with information of the technical benefit derivable from the incorporation of FACTS controllers within the system. This purpose of this paper is the incorporation of FACTS controllers, STATCOM, HVDC-VSC and UPFC within the Nigerian power system and to show the improvements in the system due to the installation of the following FACTS controllers for power flow analysis. Power flow analysis of the Nigerian power system with the incorporation of FACTS controllers requires efficient models of FACTS controllers.

The Power Injection Model (PIM) of the three FACTS controllers considered in this work which had been well tested in standard system with good results have been adopted and presented with the voltage expressed in rectangular form [3],[4],[5][6]. In order to carry out the power flow analysis of the Nigerian power system with the incorporation of FACTS controllers an existing Newton-Raphson MATLAB based program was suitably extended to include the FACTS controllers; STATCOM, HVDC-VSC and UPFC.

2. Power Flow of Facts Controllers

The power flow models of the three FACTS controllers and their linearised equations are presented.

2.1 Static Synchronous Compensator (STATCOM) power flow model

The STATCOM is a FACTS controller based on voltage sourced converter (VSC). A VSC generate a synchronous voltage of fundamental frequency and controllable magnitude and phase angle [3]. If a VSC is shunt-connected to a system via a coupling transformer as shown in Fig. 1, Modelling of STATCOM controller within the Newton-Raphson method in rectangular co-ordinates is carried out as follows:
The Thevenin equivalent circuit representing the fundamental frequency operation of the switched-mode voltage sourced converter and its transformer is shown in Figure 1.

$$V_{STC} = V_k + Z_{SC} I_{STC} \tag{1}$$

is expressed in Norton equivalent form

$$I_{STC} = I_N - Y_{SC} V_k \tag{2}$$

where

$$I_N = Y_{SC} V_{STC}$$

In these expressions, V_{STC} represents the voltage source inverter while I_{STC} is its associated current. Also, Y_{SC} is the transformer's short-circuiting admittance.

The STATCOM voltage injection V_{STC} bound constraints is as follows:

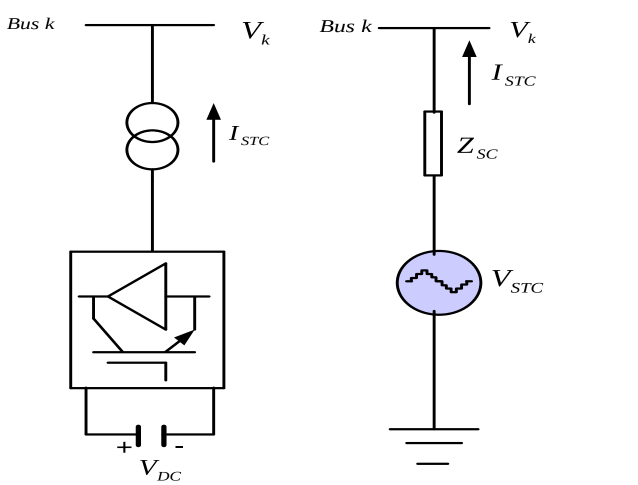
$$V_{STC\ \min} \le V_{STC} \le V_{STC\ \max} \tag{3}$$

Where $V_{STC\ \min}$ and $V_{STC\ \max}$ are the STATCOM's minimum and maximum voltages.

The current expression in (2) is transformed into a power expression by the VSC and power injected into bus k as shown in equations (4) and (5) respectively.

$$S_{STC} = V_{STC} I_{STC}^* = V_{STC}^2 Y_{SC}^* - V_{STC} Y_{SC}^* V_k^* \tag{4}$$

$$S_k = V_k I_{STC}^* = V_{STC} Y_{SC}^* V_k^* - V_k^2 Y_{SC}^* \tag{5}$$



(a) STATCOM schematic diagram (b) STATCOM equivalent circuit

Figure 1: Thevenin equivalent circuit diagram

Using the rectangular coordinate representation,

$$V_k = e_k + jf_k$$

$$V_{STC} = e_{STC} + jf_{STC}$$

$$|V_{STC}| = \left(e_{STC}^2 + f_{STC}^2\right)^{\frac{1}{2}}$$

$$\delta_{STC} = \tan^{-1}\left(\frac{f_{STC}}{e_{STC}}\right)$$

where $|V_{STC}|$ and δ_{STC} are the STATCOM voltage magnitude and angle respectively.

The active and reactive powers for the STATCOM and node k respectively are:

$$P_{STC} = G_{SC}\left\{\left(e_{STC}^2 + f_{STC}^2\right) - \left(e_{STC}e_k + f_{STC}f_k\right)\right\} + B_{SC}\left(e_{STC}f_k - f_{STC}e_k\right) \qquad (6)$$

$$Q_{STC} = G_{SC}\left(e_{STC}e_k - f_{STC}e_k\right) + B_{SC}\left(-e_{STC}^2 - f_{STC}^2 + e_{STC}e_k + f_{STC}f_k\right) \qquad (7)$$

and

$$P_k = G_{SC}\left\{e_k^2 + f_k^2 - \left(e_k e_{STC} + f_k f_{STC}\right)\right\} + B_{SC}\left(e_k f_{STC} - f_k e_{STC}\right) \qquad (8)$$

$$Q_k = G_{SC}\left(e_k f_{STC} - f_k e_{STC}\right) + B_{SC}\left\{\left(e_k f_{STC} + f_k e_{STC}\right) - \left(e_k^2 + f_k^2\right)\right\} \qquad (9)$$

- **Linearised Power Equations**

The linearised set of equations, assuming that the STATCOM is connected to bus k of the network and that the active power of the VSC is constant, may be given by

$$\begin{bmatrix} \Delta P_k \\ \Delta|V_k|^2 \\ \Delta P_{STC} \\ \Delta Q_{STC} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_k}{\partial e_k} & \frac{\partial P_k}{\partial f_k} & \frac{\partial P_k}{\partial e_{STC}} & \frac{\partial P_k}{\partial f_{STC}} \\ \partial|V_k|^2 & \frac{\partial|V_k|^2}{\partial f_k} & 0 & 0 \\ \frac{\partial P_{STC}}{\partial e_k} & \frac{\partial P_{STC}}{\partial f_k} & \frac{\partial P_{STC}}{\partial e_{STC}} & \frac{\partial P_{STC}}{\partial f_{STC}} \\ \frac{\partial Q_{STC}}{\partial e_k} & \frac{\partial Q_{STC}}{\partial f_k} & \frac{\partial Q_{STC}}{\partial e_{STC}} & \frac{\partial Q_{STC}}{\partial f_{STC}} \end{bmatrix} \begin{bmatrix} \Delta e_k \\ \Delta f_k \\ \Delta e_{STC} \\ \Delta f_{STC} \end{bmatrix} \qquad [10]$$

2.2 HVDC-VSC Power Flow Model

The HVDC-VSC consists of two VSC stations, one operating as a rectifier and the other as an inverter. The two converters are connected either back-to-back (B-T-B) or joined together by a DC cable, depending on the application [3]. For the purpose of fundamental frequency analysis, each converter station can be represented by a complex voltage source V_{VR} behind the transformer's reactance X (impedance Z) linked together by active power constraint equation. Hence, a schematic representation and equivalent circuit shown in Figure 2(a) and 2(b) are used to derive the mathematical model of the HVDC-VSC in rectangular form for inclusion in the power flow Newton-Raphson method.

The complex voltage sources representing the two VSC stations in the HVDC-VSC link are:

$$V_{VR1} = |V_{VR1}|\angle\delta_{VR1} = |V_{VR1}|\left(\cos\delta_{VR1} + j\sin\delta_{VR1}\right) = e_{VR1} + jf_{VR1} \qquad (11)$$

$$V_{VR2} = |V_{VR2}|\angle\delta_{VR2} = |V_{VR2}|\left(\cos\delta_{VR2} + j\sin\delta_{VR2}\right) = e_{VR2} + jf_{VR2} \qquad (12)$$

The voltage sources have the following voltage magnitudes and phase angle limits:

$0 \le V_{VR1} \le V_{VR1\max}$; $0 \le \delta_{VR1} \le 2\pi$; $0 \le V_{VR2} \le V_{VR2\max}$; $0 \le \delta_{VR2} \le 2\pi$.

The constraining power equation for the back-to-back HVDC-VSC, i.e. $R_{DC} = 0$ is given by

$$\mathrm{Re}\{V_{VR1}I_{VR1}^{*}+V_{VR2}I_{VR2}^{*}\}=0 \tag{13}$$

and for the case when both VSC stations are linked by a DC cable, i.e. $R_{DC}>0$, *then*

$$\mathrm{Re}\{V_{VR1}I_{VR1}^{*}+V_{VR2}I_{VR2}^{*}+P_{DC,loss}\}=0 \tag{14}$$

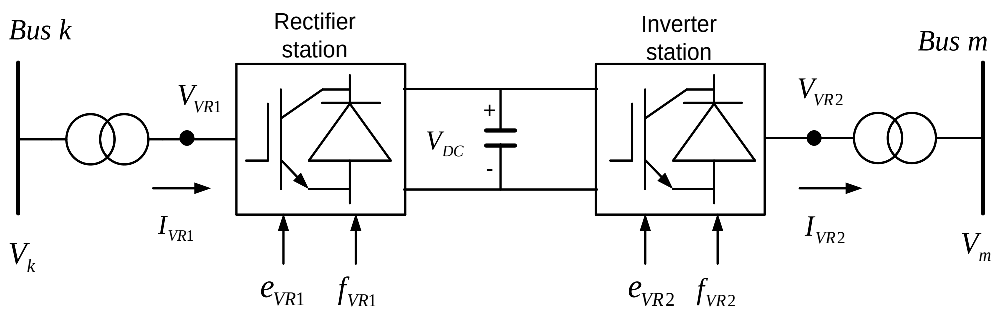


Figure 2(a): HVDC based VSC system

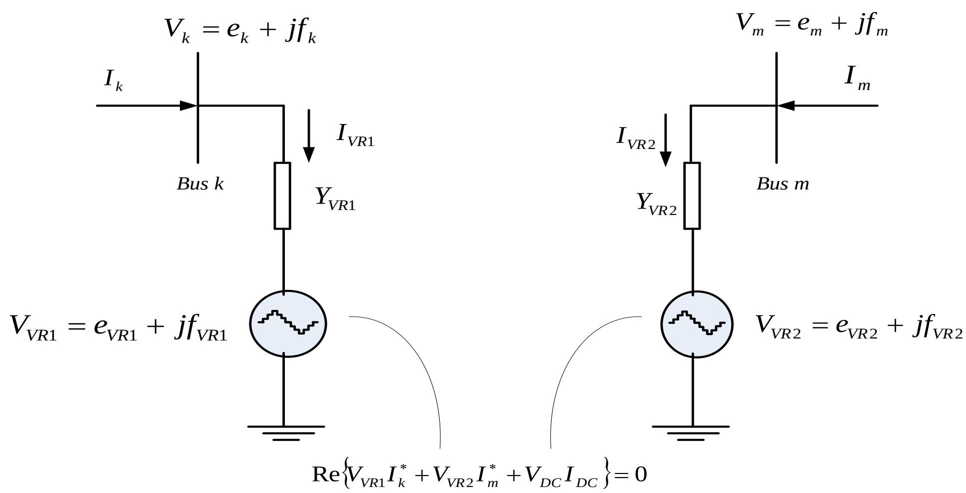


Figure 2 (b) HVDCbased VSC equivalent circuit

The power flows from the station connected at bus k (rectifier) to the station connected at bus m (inverter), the power injected at bus *k* are

$$P_k = G_{VR1}\{e_k^2+f_k^2-(e_{VR1}+f_{VR1})\}+B_{VR1}(f_{VR1}e_k-e_{VR1}f_k) \tag{15}$$

$$Q_k = G_{VR1}(f_{VR1}e_k-e_{VR1}f_k)+B_{VR1}\{-(e_k^2+f_k^2)+e_{VR1}e_k+f_{VR1}f_k\} \tag{16}$$

The power flows into the rectifier are described by the following equations:

$$P_{VR1} = -G_{VR1}\{-(e_{VR1}^2+f_{VR1}^2)+e_{VR1}e_k+f_{VR1}f_k\}+B_{VR1}(e_{VR1}f_k-f_{VR1}e_k) \tag{17}$$

$$Q_{VR1} = G_{VR1}(e_{VR1}f_k-f_{VR1}e_k)+B_{VR1}\{-(e_{VR1}^2+f_{VR1}^2)+e_{VR1}e_k+f_{VR1}f_k\} \tag{18}$$

The power equations for the node *m* and for the inverter are simply obtained by exchanging the subscripts *k* and V_{VR1} for *m* and V_{VR2}, respectively.

In addition, one more equation is required to represent the power constraint given in the form of either equation (13) or (14), depending on the application. In case of full HVDC-VSC, the relevant power equation is:

$$P_{HVDC} = P_{VR1}+P_{VR2}+P_{DC}=0 \tag{19}$$

- **Linearised System of Equations**

A Newton power flow algorithm with simultaneous solution of power flow constraints and power flow control constraints of the HVDC-VSC may be represented by

$$[f(X)] = [J][\Delta X] \tag{20}$$

where

$$[f(X)] = \left[\Delta P_k \ \Delta Q_k \ \Delta P_m \ \Delta Q_m \ \Delta P_{VR1} \ \Delta Q_{VR1} \ \Delta P_{HVDC} \ \Delta |V_m|^2 \right]^T$$

$$[\Delta X] = \left[\Delta e_k \ \Delta f_k \ \Delta e_m \ \Delta f_m \ \Delta e_{VR1} \ \Delta f_{VR1} \ \Delta e_{VR2} \ \Delta f_{VR2} \right]^T$$

$$[J] = \begin{bmatrix}
\frac{\partial P_k}{\partial e_k} & \frac{\partial P_k}{\partial f_k} & \frac{\partial P_k}{\partial e_m} & \frac{\partial P_k}{\partial f_m} & \frac{\partial P_k}{\partial e_{VR1}} & \frac{\partial P_k}{\partial f_{VR1}} & 0 & 0 \\
\frac{\partial Q_k}{\partial e_k} & \frac{\partial Q_k}{\partial f_k} & \frac{\partial Q_k}{\partial e_m} & \frac{\partial Q_k}{\partial f_m} & \frac{\partial Q_k}{\partial e_{VR1}} & \frac{\partial Q_k}{\partial f_{VR1}} & 0 & 0 \\
\frac{\partial P_m}{\partial e_k} & \frac{\partial P_m}{\partial f_k} & \frac{\partial P_m}{\partial e_m} & \frac{\partial P_m}{\partial f_m} & 0 & 0 & \frac{\partial P_m}{\partial e_{VR2}} & \frac{\partial P_m}{\partial f_{VR2}} \\
\frac{\partial Q_m}{\partial e_k} & \frac{\partial Q_m}{\partial f_k} & \frac{\partial Q_m}{\partial e_m} & \frac{\partial Q_m}{\partial f_m} & 0 & 0 & \frac{\partial Q_m}{\partial e_{VR2}} & \frac{\partial Q_m}{\partial f_{VR2}} \\
\frac{\partial P_{VR1}}{\partial e_k} & \frac{\partial P_{VR1}}{\partial f_k} & \frac{\partial P_{VR1}}{\partial e_m} & \frac{\partial P_{VR1}}{\partial f_m} & \frac{\partial P_{VR1}}{\partial e_{VR1}} & \frac{\partial P_{VR1}}{\partial f_{VR1}} & 0 & 0 \\
\frac{\partial Q_{VR1}}{\partial e_k} & \frac{\partial Q_{VR1}}{\partial f_k} & \frac{\partial Q_{VR1}}{\partial e_m} & \frac{\partial Q_{VR1}}{\partial f_m} & \frac{\partial Q_{VR1}}{\partial e_{VR1}} & \frac{\partial Q_{VR1}}{\partial f_{VR1}} & 0 & 0 \\
\frac{\partial P_{HVDC}}{\partial e_k} & \frac{\partial P_{HVDC}}{\partial f_k} & \frac{\partial P_{HVDC}}{\partial e_m} & \frac{\partial P_{HVDC}}{\partial f_m} & \frac{\partial P_{HVDC}}{\partial e_{VR1}} & \frac{\partial P_{HVDC}}{\partial f_{VR1}} & \frac{\partial P_{HVDC}}{\partial e_{VR2}} & \frac{\partial P_{HVDC}}{\partial f_{VR2}} \\
0 & 0 & \frac{\partial |V_m|^2}{\partial e_m} & \frac{\partial |V_m|^2}{\partial f_m} & 0 & 0 & 0 & 0
\end{bmatrix} \tag{21}$$

2.3 Unified Power Flow Controller (UPFC) Power Flow Model

The UPFC can provide simultaneous control of all basic power system parameters (transmission voltage, impedance and phase angle) and dynamic compensation of AC system. The controller can fulfil functions of reactive shunt compensation, series compensation and phase shifting meeting multiple control objectives [3]. A schematic representation of a UPFC is shown in Figure 3(a).The output voltage of the series converter is added to the AC terminal voltage V_0 via the series connected coupling transformer. The injected voltage V_{CR} acts as an AC series voltage source, changing the effective sending-end voltage as seen from node *m*. The product of the transmission line current I_m and the series voltage source V_{CR}, determines the active and reactive power exchanged between the series converter and the AC system. The UPFC equivalent circuit shown in Figure 3(b) is used to derive the steady-state model in rectangular form. The equivalent circuit consists of two ideal voltage sources representing the fundamental Fourier series component of the switched voltage waveforms at the AC converter terminals. The ideal voltages sources are:

$$V_{VR} = |V_{VR}|(\cos\delta_{VR} + j\sin\delta_{VR}) = e_{VR} + jf_{VR} \tag{22}$$

$$V_{CR} = |V_{CR}|(\cos\delta_{CR} + j\sin\delta_{CR}) = e_{CR} + jf_{VR} \tag{23}$$

where $|V_{VR}|$ and δ_{VR} are the controllable magnitude $(|V_{VR\min}| \le |V_{VR}| \le |V_{VR\max}|)$ and angle $(0 \le \delta_{VR} \le 2\pi)$ of the voltage source representing the shunt converter. The magnitude $|V_{CR}|$ and angle δ_{CR} of the voltage source of the series converter are controlled between limits $(|V_{CR\min}| \le |V_{CR}| \le |V_{CR\max}|)$ and $(0 \le \delta_{CR} \le 2\pi)$, respectively.

Based on the equivalent circuit shown in Figure 3, the power equations for the UPFC are as follows:

At the sending-end node *k*:

$$S_k = P_k + jQ_k = \left(V_k^2 - V_k V_{CR}^* - V_k V_m^*\right)Y_{CR}^* + \left(V_k^2 - V_k V_{VR}^*\right)Y_{VR}^* \qquad (24)$$

After performing some complex operations, the following active and reactive power equations are obtained for bus k ,

$$\begin{aligned} P_k = {} & \left(e_k^2 + f_k^2\right)G_{kk} + G_{km}\left(e_k e_m + f_k f_m + e_k e_{CR} + f_k f_{CR}\right) \\ & + B_{km}\left(f_k e_m - e_k f_m + f_k e_{CR} - e_k f_{CR}\right) + G_{VR}\left(e_k e_{VR} - f_k f_{VR}\right) + B_{VR}\left(f_k e_{VR} - e_k f_{VR}\right) \end{aligned} \qquad (25)$$

$$\begin{aligned} Q_k = {} & -\left(e_k^2 + f_k^2\right)B_{kk} + G_{km}\left(f_k e_m - e_k f_m + f_k e_{CR} - e_k f_{CR}\right) \\ & - B_{km}\left(e_k e_m + f_k f_m + e_k e_{CR} + f_k f_{CR}\right) \\ & + G_{VR}\left(f_k e_{VR} - e_k f_{VR}\right) - B_{VR}\left(e_k e_{VR} + f_k f_{VR}\right) \end{aligned} \qquad (26)$$

At the receiving-end node *m*:

$$S_m = P_m + jQ_m = V_m\left(V_m^* + V_{CR}^* - V_k^*\right)Y_{CR}^* \qquad (27)$$

Similarly, the receiving-end node active and reactive power equations are:

$$\begin{aligned} P_m = {} & \left(e_m^2 + f_m^2\right)G_{mm} + G_{mk}\left(e_m e_k + f_m f_k\right) + B_{mk}\left(f_m e_k - e_m f_k\right) \\ & + G_{mm}\left(e_m e_{CR} + f_m f_{CR}\right) + B_{mm}\left(f_m e_{CR} - e_m f_{CR}\right) \end{aligned} \qquad (28)$$

$$\begin{aligned} Q_m = {} & -\left(e_m^2 + f_m^2\right)B_{mm} + G_{mk}\left(f_m e_k - e_m f_k\right) - B_{mk}\left(f_m f_k + e_m e_k\right) \\ & + G_{mm}\left(f_m e_{CR} - e_m f_{CR}\right) - B_{mm}\left(e_m e_{CR} + f_m f_{CR}\right) \end{aligned} \qquad (29)$$

Series converter power:

$$S_{CR} = V_{CR} I_{CR}^* = P_{CR} + jQ_{CR} = V_{CR}\left(V_m^* + V_{CR}^* - V_k^*\right)Y_{CR}^* \qquad (30)$$

$$\begin{aligned} P_{CR} = {} & \left(e_{CR}^2 + f_{CR}^2\right)G_{mm} + G_{km}\left(e_{CR} e_k + f_{CR} f_k\right) + B_{km}\left(f_{CR} e_k - e_{CR} f_k\right) \\ & + G_{mm}\left(e_{CR} e_m + f_{CR} f_m\right) + B_{mm}\left(f_{CR} e_m - e_{CR} f_m\right) \end{aligned} \qquad (31)$$

$$\begin{aligned} Q_{CR} = {} & -\left(e_{CR}^2 + f_{CR}^2\right)B_{mm} + G_{km}\left(f_{CR} e_k - e_{CR} f_k\right) - B_{km}\left(f_{CR} f_k + e_{CR} e_k\right) \\ & + G_{mm}\left(f_{CR} e_m - e_{CR} f_m\right) - B_{mm}\left(f_{CR} f_m + e_{CR} e_m\right) \end{aligned} \qquad (32)$$

Shunt converter:

$$S_{VR} = V_{VR} I_{VR}^* = P_{VR} + jQ_{VR} = V_{VR}\left(V_k^* - V_{VR}^*\right)Y_{VR}^* \qquad (33)$$

The shunt converter active and reactive power equations are:

$$P_{VR} = -\left(e_{VR}^2 + f_{VR}^2\right)G_{VR} + G_{VR}\left(e_{VR} e_k + f_{VR} f_k\right) + B_{VR}\left(f_{VR} e_k - e_{VR} f_k\right) \qquad (34)$$

$$Q_{VR} = \left(e_{vR}^2 + f_{VR}^2\right)B_{VR} + G_{VR}\left(f_{VR} e_k - e_{VR} f_k\right) - B_{VR}\left(e_{VR} e_k + f_{VR} f_k\right) \qquad (35)$$

where

$$Y_{kk} = G_{kk} + jB_{kk} = y_{CR} + y_{VR}$$
$$Y_{mm} = G_{mm} + jB_{mm} = y_{CR}$$

The series converter active and reactive power equations are:

$$Y_{km} = Y_{mk} = G_{km} + jB_{km} = -y_{CR}$$
$$Y_{VR} = G_{VR} + jB_{VR} = -y_{VR}$$

Assuming a free loss converter operation, the UPFC neither absorbs nor injects active power with respect to the AC system. The DC link voltage, V_{dc} , remains constant. The active power associated with the series converter becomes the DC power $V_{dc} I_2$. The shunt converter must supply an equivalent amount of DC power to maintain V_{dc} constant. Hence, the active power supplied to the shunt converter, P_{VR} , must satisfy the active power demanded by the series converter, P_{CR} , i.e.

$$P_{bb} = P_{VR} + P_{CR} = 0 \qquad (36)$$

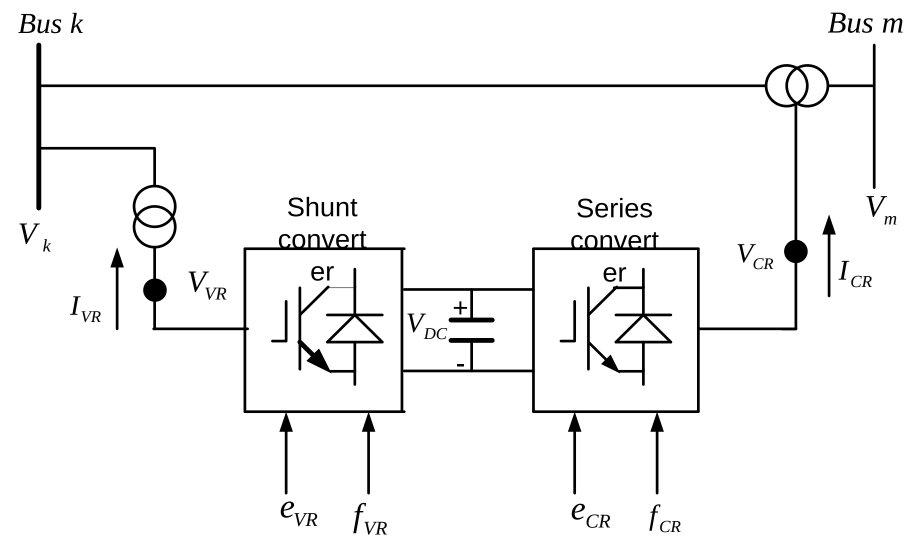


Figure 3 (a) A Schematic representation of the UPFC controller

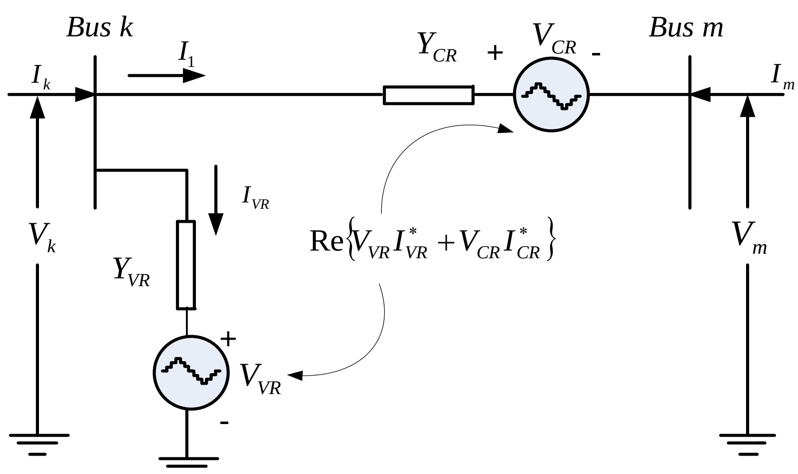


Figure 3 (b) Equivalent circuit of the Unified Power Flow Controller

- **Linearised system of equations**

The UPFC linearised power equations are combined with the linearised system of equations corresponding to the rest of the network,

$$[f(x)] = [J][\Delta X] \qquad (37)$$

where

$$[f(x)] = \left[\Delta P_k \ \Delta P_m \ \Delta Q_k \ \Delta Q_m \ \Delta P_{mk} \ \Delta Q_{mk} \ \Delta P_{bb} \ \Delta |V_k|^2\right]^T \qquad (38)$$

$$\Delta|V_k| = |V_{k(specified)}| - |V_{k(calculated)}| \qquad (39)$$

ΔP_{bb} is the power mismatch given by equation (38) and the superscript T indicates transposition. $[X]$ is the solution vector and $[J]$ is the Jacobian matrix.

For the case when the UPFC controls voltage magnitude at the AC shunt converter terminal (node *k*), active power flowing from node *m* to node *k* and reactive power injected at node *m* , and assuming that node *m* is PQ-type, the solution vector and Jacobian matrix are,

$$\left[\Delta X\right]=\left[\Delta e_k\ \Delta e_m\ \Delta f_k\ \Delta f_m\ \Delta e_{VR}\ \Delta f_{VR}\ \Delta e_{CR}\ \Delta f_{VR}\right]^T \tag{40}$$

$$\left[J\right]=\begin{bmatrix}
\frac{\partial P_k}{\partial e_k} & \frac{\partial P_k}{\partial e_m} & \frac{\partial P_k}{\partial f_k} & \frac{\partial P_k}{\partial f_m} & \frac{\partial P_k}{\partial e_{VR}} & \frac{\partial P_k}{\partial f_{VR}} & \frac{\partial P_k}{\partial e_{CR}} & \frac{\partial P_k}{\partial f_{CR}} \\
\frac{\partial P_m}{\partial e_k} & \frac{\partial P_m}{\partial e_m} & \frac{\partial P_m}{\partial f_k} & \frac{\partial P_m}{\partial f_m} & 0 & 0 & \frac{\partial P_m}{\partial e_{CR}} & \frac{\partial P_m}{\partial f_{CR}} \\
\frac{\partial Q_k}{\partial e_k} & \frac{\partial Q_k}{\partial e_m} & \frac{\partial Q_k}{\partial f_k} & \frac{\partial Q_k}{\partial f_m} & \frac{\partial Q_k}{\partial e_{VR}} & \frac{\partial Q_k}{\partial f_{VR}} & \frac{\partial Q_k}{\partial e_{CR}} & \frac{\partial Q_k}{\partial f_{CR}} \\
\frac{\partial Q_m}{\partial e_k} & \frac{\partial Q_m}{\partial e_m} & \frac{\partial Q_m}{\partial f_k} & \frac{\partial Q_m}{\partial f_m} & 0 & 0 & \frac{\partial Q_m}{\partial e_{CR}} & \frac{\partial Q_m}{\partial f_{CR}} \\
\frac{\partial P_{mk}}{\partial e_k} & \frac{\partial P_{mk}}{\partial e_m} & \frac{\partial P_{mk}}{\partial f_k} & \frac{\partial P_{mk}}{\partial f_m} & 0 & 0 & \frac{\partial P_{mk}}{\partial e_{CR}} & \frac{\partial P_{mk}}{\partial f_{CR}} \\
\frac{\partial Q_{mk}}{\partial e_k} & \frac{\partial Q_{mk}}{\partial e_m} & \frac{\partial Q_{mk}}{\partial f_k} & \frac{\partial Q_{mk}}{\partial f_m} & 0 & 0 & \frac{\partial Q_{mk}}{\partial e_{CR}} & \frac{\partial Q_{mk}}{\partial f_{CR}} \\
\frac{\partial P_{bb}}{\partial e_k} & \frac{\partial P_{bb}}{\partial e_m} & \frac{\partial P_{bb}}{\partial f_k} & \frac{\partial P_{bb}}{\partial f_m} & \frac{\partial P_{bb}}{\partial e_{VR}} & \frac{\partial P_{bb}}{\partial f_{VR}} & \frac{\partial P_{bb}}{\partial e_{CR}} & \frac{\partial P_{bb}}{\partial f_{CR}} \\
\frac{\partial \left|V_k\right|^2}{\partial e_k} & 0 & \frac{\partial \left|V_k\right|^2}{\partial f_k} & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{41}$$

3. Implementation

A MATLAB based program for the power flow analysis of electrical power systems was suitably extended to include steady-state models of the three FACTS controllers; STATCOM, HVDC-VSC and UPFC. The program will henceforth be referred to Flexible Alternating Current Transmission System Power Flow (FACTSPF). The procedure for power flow solution by the Newton-Raphson method without and with FACTS controllers is shown in flowchart of Figure 4.

3.1 Power flow analysis Nigerian transmission power system

The single line diagram (Figure 5) of the Nigerian 330kV network consists of seven generating stations, twenty-four load stations and thirty-nine transmission lines. The system may be divided into three major sections: - North, South-East and the South-West sections. The North is connected to the South through one triple circuit lines between Jebba and Osogbo while the West is linked to the East through one transmission line from Osogbo to Benin and one double circuit line from Ikeja to Benin. The line diagram and data of the Nigerian transmission system were sourced from the National Control Centre of Power Holding Company of Nigeria, Osogbo, Nigeria.

Power flow analysis of the Nigerian transmission system was performed using FACTSPF. The power flow results of the Nigerian 24-bus 330kV transmission system are shown in Table 1. It can be observed that the voltage magnitude at buses 16 (Gombe) and 22 (Kano) are lower than the acceptable limit of ±10% for the Nigerian 330kV transmission system [7]. This confirms results of earlier studies on the system as reported in [8], [9], [10]. Also the voltage magnitude at bus 14 (New-Haven) is very close to the lower limit and disproportionate power flows in some of the system transmission lines as shown alighted in Table 2. The system has a high total active power loss of 89.68MW.

In order to alleviate the power system problems of voltage limit violations, and disproportionate power flows and high active power loss a solution method of incorporation of FACTS controllers into the existing power system is investigated in this work.

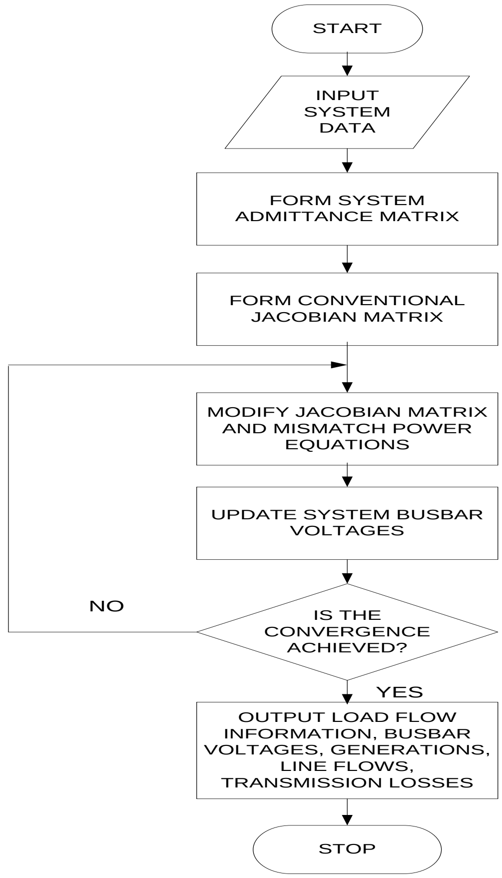


Figure 4: Flowchart for Power Flow solution by Newton-Raphson with FACTS controllers

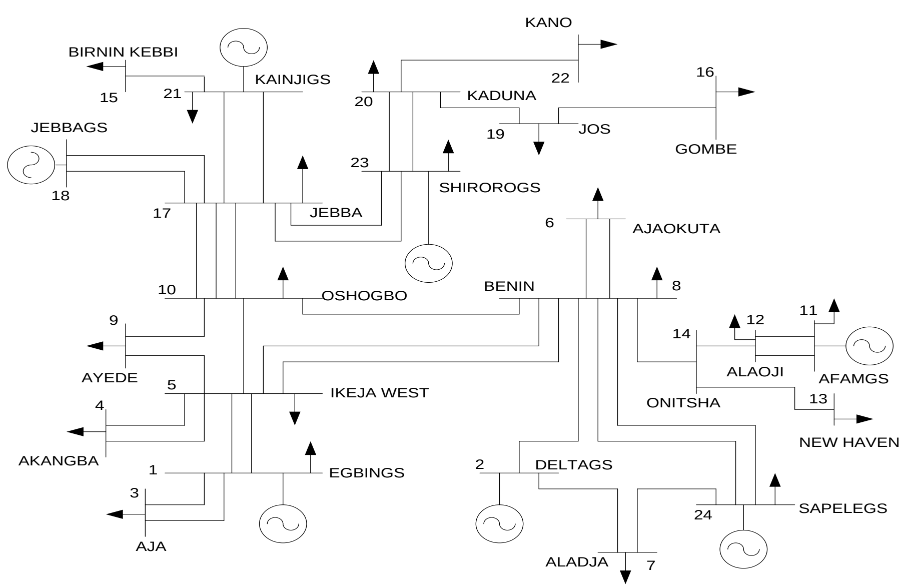


Figure 5: 24-bus 330kV Nigerian transmission system
Source: (National Control Centre, Power Holding Company of Nigerian, 2007)

Table 1: Power flow results of 24-bus system

Bus No	Bus Type	Bus Name	FACTSPF Bus Voltage Magnitude (p.u.)	FACTSPF Bus Voltage Phase angle (deg.)
1	Swing	EGBINGS	1.050	0.00
2	PV	DELTAGS	1.050	-1.14
3	PQ	AJA	1.045	-0.28
4	PQ	AKANGBA	0.988	-5.64
5	PQ	IKEJA-WEST	0.996	-5.19
6	PQ	AJAOKUTA	1.054	-6.99
7	PQ	ALADJA	1.046	-2.71
8	PQ	BENIN	1.034	-6.63
9	PQ	AYEDE	0.974	-7.79
10	PQ	OSHOGBO	1.026	-4.93
11	PV	AFAMGS	1.050	-17.27
12	PQ	ALAOJI	1.033	-17.89
13	**PQ**	**N-HAVEN**	**0.929**	**-18.89**
14	PQ	ONITSHA	0.971	-16.09
15	PQ	B-KEBBI	1.010	-3.97
16	**PQ**	**GOMBE**	**0.866**	**-31.68**
17	PQ	JEBBA	1.050	-1.61
18	PV	JEBBAGS	1.050	-1.35
19	PQ	JOS	0.948	-24.01
20	PQ	KADUNA	0.999	-16.67
21	PV	KAINJI	1.050	1.55
22	**PQ**	**KANO**	**0.880**	**-24.88**
23	PV	SHIRORO	1.050	-12.22
24	PV	SAPELE	1.050	-5.12

Table 2: Line flow and losses of Nigerian 330kV 24-bus transmission system

From Bus	To Bus	Active power flow MW	Reactive power flow Mvar	Complex power flow MVA	Active power loss MW	Reactive power loss Mvar
1	3	137.36	97.59	168.49	0.16	-5.32
1	3	137.36	97.59	168.49	0.16	-5.32
1	**5**	**573.43**	**261.18**	**630.11**	**8.43**	**39.87**
1	**5**	**573.43**	**261.18**	**630.11**	**8.43**	**39.87**
2	**8**	**330.83**	**-0.59**	**330.83**	**4.29**	**-14.03**
2	**7**	**339.17**	**3.61**	**339.19**	**1.25**	**-3.64**
4	5	-72.35	-129.25	215.43	0.33	-4.22
4	5	-72.35	-129.25	215.43	0.33	-4.22
5	8	24.63	-104.06	106.93	0.34	-110.80
5	8	24.63	-104.06	106.93	0.34	-110.80
5	9	113.71	14.37	114.61	0.80	-45.80
5	10	-11.50	-88.60	89.34	0.17	-99.91
6	8	-6.90	-5.15	8.61	0.10	-82.80
6	8	-6.90	-5.15	8.61	0.10	-82.80
7	24	241.42	-65.15	250.05	1.39	-16.80
8	**14**	**425.90**	**109.42**	**439.73**	**10.12**	**21.72**
8	10	-40.15	-35.58	53.58	0.18	-103.36
8	24	-203.97	-89.56	222.76	0.89	-14.71
8	24	-203.97	-89.56	222.76	0.89	-14.71
9	10	-162.89	-146.63	219.17	2.00	-30.04
10	17	-139.36	-62.80	152.86	1.18	-57.15
10	17	-139.36	-62.80	152.86	1.18	-57.15
10	17	-139.36	-62.80	152.86	1.18	-57.15
11	12	189.25	212.74	284.74	0.76	-5.05
11	12	189.25	212.74	284.74	0.76	-5.05
12	14	-50.01	115.39	125.76	1.37	-50.44
13	14	-177.90	-133.40	222.36	1.90	-18.27
15	21	-114.50	-85.90	143.14	1.64	-116.92
16	19	-130.60	-97.90	163.22	3.14	-73.54
17	18	-247.39	28.16	248.99	0.11	-1.04
17	18	-247.39	28.16	248.99	0.11	-1.04
17	**23**	**280.01**	**-63.90**	**287.21**	**6.84**	**-54.35**
17	**23**	**280.01**	**-63.90**	**287.21**	**6.84**	**-54.35**
17	21	-248.94	23.16	250.01	1.85	-21.25
17	21	-248.94	23.16	250.01	1.85	-21.25
19	20	-204.04	-77.06	218.10	3.91	-47.36
20	**22**	**205.98**	**116.52**	**236.65**	**6.18**	**-33.38**
20	**23**	**-303.46**	**-145.46**	**336.52**	**4.11**	**-8.89**
20	**23**	**-303.46**	**-145.46**	**336.52**	**4.11**	**-8.89**
Total					**89.68**	**-1340.30**

3.2 FACTS Controllers Incorporated in 24-bus Transmission System

Test case studies were conducted in which FACTS controllers were incorporated in the system to evaluate their performances in alleviating the voltage limit violations and power flow problems associated with the Nigerian 24-bus 330kV transmission system. The test cases are:

1. Base case without the FACTS controllers.
2. Similar to test case 1 except that three STATCOMs were installed at buses 13 (New-Haven), 16 (Gombe), and 22 (Kano) to control their voltage magnitudes to a reference value of 0.95p.u.

3. Similar to test case 1 except that HVDC-VSC (BTB) was installed to control the voltage magnitude at Bus 5(Ikeja-west) to 1.0 p.u. Active power flow from bus 5 to bus 9 controlled by +5.5% (6.24MW) of the base case while the reactive power was regulated at 9.0Mvar.
4. Similar to test case 1 except that UPFC was installed between bus 5(Ikeja-west) and bus 9 (Ayede) to control Ikeja-West bus voltage magnitude at 1.00p.u., and to regulate active and reactive power flow from bus 9 (Ayede) to bus 5 (Ikeja-west) at 136.445MW and14.37Mvar respectively.

The power flow results of the above test cases are summarized in Tables 3, 4 and Figure 6. For test case 2 the three installed STATCOMs controlled the voltage magnitude at their respective buses to the specified values by injecting reactive powers, 34.48Mvar, 20.20Mvar, and 53.72Mvar (Table 4). More reactive power was therefore available in the system that resulted in the redistribution of power flows with a reduction in system active power loss to a value of 83.65MW (6.68% reduction). The buses adjacent to the STATCOM buses experienced improvement in their voltage magnitudes as shown in Table 3(a). The STATCOM state variables are shown in Table 4. This test case reveals that the STATCOM has very little influence on the active power flows through the system transmission lines as their values remained almost the same when compared to the base test case 1.

Table 3(b) shows bus voltage magnitudes of HVDC-VSC buses and the two adjacent buses for base case (test case 1) and test case 3 when the HVDC-VSC was installed in the system. The installation of the HVDC-VSC resulted in little improvement in the voltage profile of the system. In order to keep bus 5 voltage at 1.0pu the sending-end converter injected reactive power of 33.95MVAR.
Table 4 shows the HVDC-VSC converters voltages, as well as the sending-end and receiving-end complex powers. The sending-end real and reactive powers are 18MW, 32.57MVAR while the receiving-end converter real and reactive powers are 17.97MW, 85.00MVAR respectively. The drop in active power is due the cable resistance.

For test case 4 the UPFC was set to simultaneously control active power flow, reactive power flow, and voltage magnitude at its terminals. Table 3(b) show the bus voltage magnitudes, phase angles for the UPFC buses and those adjacent to UPFC terminals. As can be observed from Table 4 the UPFC was able to regulate Ikeja-West bus voltage at 1.00p.u.by injecting a reactive power of 61.74Mvar. The changes in bus voltage angles was due to the redistribution of active power in the transmission lines as a result of active power controlled between bus 9 (Ayede) and bus 5 (Ikeja-West) at 136.44MW (Table 4). The power active power loss without and with the FACTS controllers are shown in Figure 6. All FACTS controllers reduce the system total active power loss. With STATCOM total loss reduces to 86.65MW (3.38% loss reduction); with HVDC-VSC total active power loss reduces to 89.15MW (0.59% loss reduction), and with UPFC total active power reduces to 88.93MW (0.84% loss reduction).

Table 4 shows the UPFC parameters. Shown in Table 5 are the power flows in lines connected to Ayede and Ikeja-West without and with UPFC installed between Ayede and Ikeja-West. From the Table 5 it can be seen that the power flow pattern through the lines changed in order to satisfy the power flow control along the Ayede- Ikeja-West transmission line. From test case 4 the UPFC had significant effects on power flows of lines directly connected to its two terminals as it performs the task of power flow control but has little effect or no effect on transmission line far away from it. It also has little effect on system various bus voltage magnitudes.

In conclusion the Newton-Raphson power flow solution of the Nigerian 330kV transmission system with the incorporation of the STATCOM, HVDC-VSC, and UPFC converges quadratically in maximum of 5 to 6 iterations for all test cases as shown in Figure 6. From test cases 2-4 it can be concluded that the FACTS controllers can be used to control bus voltage magnitude, active, and reactive power flow in the Nigerian transmission system. Table 5 shows that the installation of FACTS controllers did not result in significant change in the active power flow of transmission lines connected to the terminals of the FACTS controllers. In other to have a significant improvement in the system voltage profile and power flows in the system many FACTS controllers will have to be incorporated, the cost of which may be prohibitive. In view of this it becomes necessary to consider reinforcement of the weak longitudinal power system with additional generation facilities and transmission line expansion to strengthen it before incorporating FACTS controllers to further improve bus voltage magnitude, active and reactive power flows through the transmission lines and reduce active power loss.

Table 3(a):Voltage magnitudes and phase angles of some buses for base and test cases 2

Bus No	Bus Name	Base Case(without STATCOM) Nodal Voltage Magnitude (p.u.)	Phase angle (deg.)	Test Case 7 (with STATCOM) Nodal Voltage Magnitude (p.u.)	Phase angle (deg.)
13	N-HAVEN	0.929	-18.89	0.950	-18.74
14	ONITSHA	0.971	-16.09	0.982	-15.95
16	**GOMBE**	**0.866**	**-31.68**	**0.950**	**-30.33**
19	**JOS**	**0.948**	**-24.01**	**0.966**	**-23.84**
20	**KADUNA**	**0.999**	**-16.67**	**1.018**	**-16.45**
22	**KANO**	**0.880**	**-24.88**	**0.950**	**-24.23**

Table 3(b): Voltage magnitudes and phase angle of some buses for base case, test cases 3 and 4

Bus No	Bus Name	Base case(without FACTS) Nodal Voltage Magnitude (p.u.)	Phase Angle (deg.)	Test case (with HVDC-VSC) Nodal Voltage Magnitude (p.u.)	Phase Angle (deg.)	Test case 10(with UPFC) Nodal Voltage Magnitude (p.u.)	Phase Angle (deg.)
4	AKANGBA	**0.987**	**-5.66**	**0.992**	**-5.64**	**0.992**	**-5.65**
5	**IKEJA-WEST**	**0.995**	**-5.21**	**1.000**	**-5.19**	**1.000**	**-5.20**
6	AJAOKUTA	1.054	-7.04	1.054	-7.02	1.055	-6.75
8	BENIN	1.034	-6.68	1.035	-6.69	1.035	-6.39
9	AYEDE	**0.972**	**-7.84**	**0.976**	**-6.98**	**0.976**	**-6.74**
10	OSHOGBO	**1.024**	**-4.99**	**1.028**	**-4.47**	**1.028**	**-4.33**
13	N-HAVEN	0.929	-18.94	0.929	-18.52	0.929	-18.64
14	ONITSHA	0.971	-16.13	0.972	-15.95	0.972	-15.83

Table 4: FACTS controllers source voltages and complex powers in 24-bus system

Test Case	FACTS CONTROLLERS	Bus	FACTS state variables Voltage Magnitude Angle (p.u.)	Phase (deg.)	FACTS complex powers Active Power (MW)	Reactive Power (Mvar)
2	STATCOM	13	0.986	-18.74	0.00	-34.48
		16	0.971	-30.33	0.00	-20.20
		22	1.007	-24.23	0.00	-53.72
3	HVDC-VSC					
	Sending-end Converter	9	1.055	-12.28	130.00	46.84
	Receiving-end Converter	5	1.046	0.19	128.60	-59.79
4	UPFC					
	Shunt Converter	5	1.0583	-5.17	136.44	-47.73
	Series Converter	9	0.1848	-98.18	-136.44	-14.37

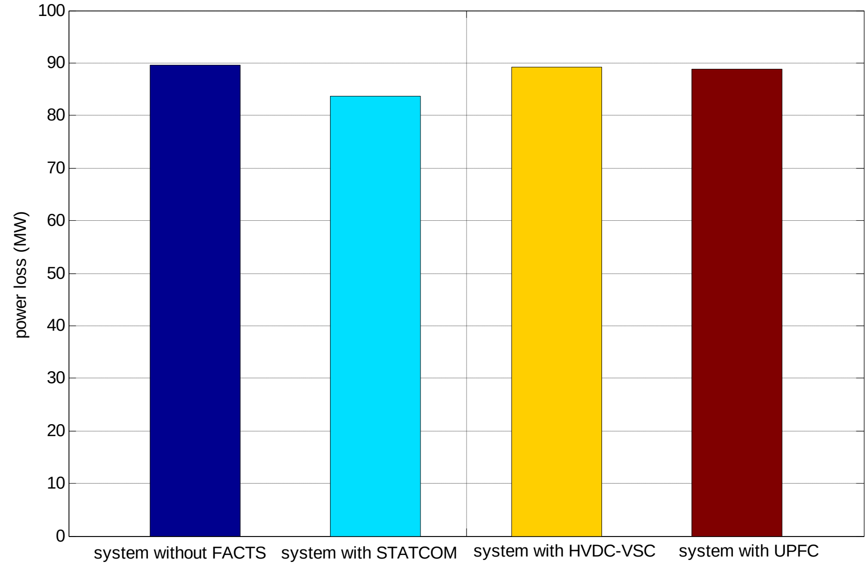


Figure 6: System active power losses of 24-bus Nigerian 330kV transmission system without and with FACTS controllers

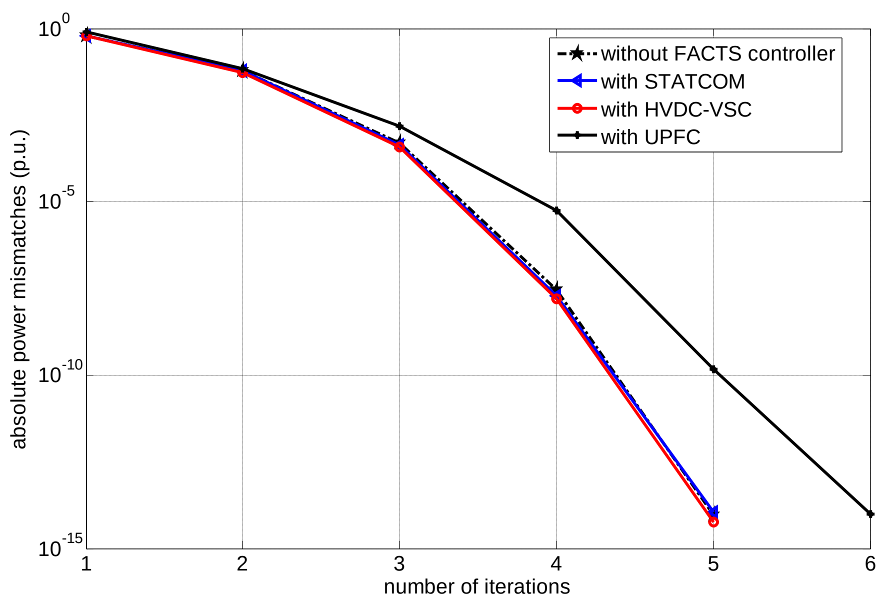


Figure 6: Absolute power mismatches as function of number of iterations for the Nigerian transmission power system

Table 5: Transmission Line Active Power Flows for Test Cases 2, 3 and 4

From Bus	To Bus	Base case Active power flow (MW)	Test case 2 STATCOM Active power flow (MW)	Test case 3 HVDC-VSC Active power flow (MW)	Test case 4 UPFC Active Power flow (MW)
5	8	24.63	23.60	15.26	20.06
5	9	113.71	111.83	161.00	136.45
5	10	-11.50	-13.44	-40.00	-25.27
8	14	425.90	424.74	425.90	425.88
8	10	-40.15	-41.00	-58.67	-49.08
9	10	-162.89	-164.74	-114.50	-140.45
10	17	-139.36	-140.92	-139.28	-139.37
12	14	-50.01	-49.93	-50.01	-50.01

4. Conclusion

Voltage Sourced Converter (VSC) based Flexible Alternating Current Transmission Systems (FACTS) controllers; STATCOM, HVDC-VSC and UPFC power injection model with the voltage expressed in rectangular form were presented in this work. Power flow analysis of the Nigerian transmission power system was carried to identify possible location for the installation of the three FACTS controllers. The steady-state models of the FACTS controllers produced algebraic equations which were combined with power system network algebraic equations and these were solved using Newton-Raphson solution technique. A MATLAB based program, Flexible Alternating Current Transmission System Power Flow (FACTSPF) for steady-state analysis of power was developed. Power flow analysis of the Nigerian transmission system conducted confirmed results of earlier works that the system is weak with voltage limit violations and high power losses. Application of the FACTS controllers resulted in the voltage profile of the system being brought within acceptable limit of $\pm 10\%$. In addition they also assisted in reducing in high power losses of system depending on type and number of FACTS controller installed. The technical benefits derivable from the incorporation of STATCOM, HVDC-VSC and UPFC in Nigerian power system have been demonstrated in this work.

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