Dynamics of non-dimensional Richardson, Wedderburn and lake Numbers applied to the neotropical reservoirs Barra Bonita and Carlos Botelho (Broa) –São Paulo- Brazil

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Abstract

The dynamics of reservoirs biodiversity can be explained by changes in the forces of functions such as climatologic and hydrologic. The aim of the present study was to determine the non-dimensional Wedderburn (W), Richardson (Ri) and Lake Numbers (LN) and their effects on the chemical and biological components of the reservoirs of Barra Bonita and Carlos Botelho (Broa). Previous data were used to show relationships between densimetric parameters biological and chemical data from the two reservoirs. Result showed occurrences of Dinophyceaean Bacillariophycea in summer, Chlorophyceae in winter, Cyanophyceae in autumn and spring, and Euglenophyceae, Cryptophyceae and Phytoflagellates the whole year. Concerning zooplankton, Thermocyclops and Mesocyclops exhibited occurrence in winter and spring. There was a positive correlation between Ri and W numbers and the nutrients. The stratified structure over the year has an influence on the development of some group of phytoplankton and chemical components, consequence of effect of non-dimensional numbers.

Keywords: Stratification, Stability, Wedderburn, Richardson, Reservoir

1. Introduction

Reservoirs are formed artificially for the purpose of producing electric energy or storing water for various uses. This involves the multiple use of a water body based on its quality. This very dynamic parameter depends on several factors including those that affect its structure, composition and quality. Heating by the sun and wind action on the surface create different horizontal and vertical currents in the liquid mass affecting its structure and distribution of chemical and biological parameters. Based on location, depth and wind action, the reservoirs show a stratified structure in a given period of the year. This structure can affect the distribution of chemical and biological components. According to Tundisi et al. (2008), in the short term, the stability in the Barra Bonita reservoir is important in controlling the mechanisms of primary productivity of phytoplankton, succession of zooplankton and distribution of nutrients. Changes in biodiversity can be explained by modifications in the forces of functions such as climatologic and hydrologic interaction with consequences on primary production, and the seasonal cycle of phytoplankton and zooplankton, beside the action of densimetric parameters of Wedderburn, Richardson and lake numbers. According to Tundisi et al. (2002), in some regions of the superficie of reservoir where there is mixing of water of different density, a large accumulation of Cyclotella meneghiana is found.
Straškraba and Tundisi (1999) noted that the change in water quality of the reservoir can occur as consequence of the increase in residence time and of the decrease in flow rate, causing strong stratification and development of a deep anoxic layer. There is a strong relation between vertical structure and colony number of *Aulacoseira* with Wedderburn number, according to Tundisi et al. (2002). Along the same lines, Chalar and Tundisi (1999) mentioned that non-dimensional numbers can also be an excellent variable in the correlation of dissolved oxygen and manganese. In this context, Matmusura-Tundisiet al. (1997) found that the retention time of the dam is a controlling factor of primary production and cycles of phytoplankton. According to Sperling (1999), the dynamics of the circulation and stratification of a water body are the main factors that regulate the distribution of substances and organisms in a liquid mass. As reported by Salencon and Thebault (1997), the circulations created are linked to the temperature and density of the fluid, which depend on hydrostatic pressure, determined by physical processes. Physical forces are not the only ones that affect Barra Bonita reservoir. The dynamics of predation on zooplankton communities by fish and the predominance of the bloom of cyanobacteria are other causes of the fluctuation of the zooplankton populations (Tundisi et al., 2008). The use or application of non-dimensional parameters allows us to understand the dynamics of processes in reservoirs and their consequences for chemical and biological processes (Tundisi et al, 2002).

Accordingly, the aim of the present study was to determine the non-dimensional Wedderburn, Richardson and lake numbers and to evaluate their effects on the chemical and biological components of the reservoirs of the hydroelectric plants Barra Bonita and Carlos Botelho (Broa).

2. **Material and methods**

For this proposal, it was necessary to find the Wedderburn number expressed as reservoir surface unit and determined by the following formula, according to Tundisi et al. (2002):

\[
W = \frac{\Delta d g' h^2}{d u^2 L}
\]

\[
\mu^2 = v \frac{d_{ar}}{d_{h2o}} C_d
\]

Where: \(\Delta d\) is difference in density between two layers
\(d\) is density of the layer (g/cm\(^3\))
\(g' h^2\) is force of maximal baroclinic pressure at the point of “upwelling”
\(g' = g \Delta \rho / \rho, g' \Delta \rho\) (difference between the water density at the surface and at thermocline depth, g/cm\(^3\))
\(h\) is the height of the mixing layer (cm)
\(u^2 L\) is the force applied by the wind on the water surface;
\(L\) is length of reservoir (m);
\(u^2\) is represented by:

\[
\mu^2 = v \frac{d_{ar}}{d_{h2o}} C_d
\]

Where: \(d_{ar}\) is air density at a given temperature and altitude (1.2 kg/m\(^3\)), \(d_{h2o}\) is water density, \(v\) is wind speed (m/s), and drag coefficient \(C_d = 0.0014\). The evaluation of the effect on the reservoir was done according to the values of the Wedderburn(W) number obtained. \(W > 10\) corresponds to a strong stratification due to weak winds. \(3 < W < 10\) characterises a weak stratification and strong winds. If \(W < 3\) winds are strong and the thermocline is close to the surface or nonexistent. *Lake number* (\(L_N\)), introduced by Imberger and Patterson (1989), was also determined. Based on the relation between the moments, it was calculated in relation to the centre of the volume and force of gravity by the formula:

\[
L_N = \frac{\left[Z - L(H)\right] A_z d_z dz (1 - [H - h]/H)}{d_{ar}^2 A_n^{1/2} (1 - [L - h]/H)}
\]

Imberger and Patterson, 1989apud (Salencon and Thebault, 1997)

Where:
\(A(z)\) corresponds to the surface of the lake at depth \(z\) and \(A_n\) the area at the surface of the lake;
\(H\) (cm) is the depth at the centre of the water mass;
\(Z\) (cm) is the total depth of the reservoir;
\(u^*\) = wind speed on surface of the reservoir;
\(d_{ar}\) = density of superficial layer;
\(h\) is the distance of the middle of the metalimnion to the surface of the lake and,
L(H) the distance to the centre of the volume or centre of the mass determined by the formula

\[ L(H) = \frac{\int_{0}^{H} A(z)dz}{\int_{0}^{H} ZA(z)dz} \]  

(4) (Salencon and Thebault, 1997)

Where Z is the maximal depth, A(z) the surface at depth (z) and H the depth up to the centre of the water mass. When \( L_N < 1 \), a condition of very weak stratification is characterised; when \( L_N > 1 \), stratification is sufficiently strong and the hypolimnion is a little turbulent, and when \( L_N = 1 \), the “upwelling” of thermocline is in contact with the water surface.

The third non-dimensional number, the Richardson (Ri) number, provides an indication of the potential resistance of the epilimnion to mixing by the wind, and can be determined by the following formula:

\[ Ri = \frac{\Delta dgh}{d_o u_*^2} \]  

(5) (Salencon and Thebault, 1997)

Where:
- \( \Delta d \) = difference in density between two layers; (g.cm\(^{-3}\))
- g = gravitational constant (9.81 m.s\(^{-1}\));
- h = height of mixing layer; (cm)
- \( d_o \) = density of superficial layer (g.cm\(^{-3}\));
- \( u_* \) = wind speed on the surface of the reservoir (m.s\(^{-1}\));

The importance of the study lies in the analysis of the effect of these non-dimensional numbers on the limnology of the reservoirs. The relations between the values obtained for the non-dimensional numbers with primary production and nutrients concentrations were evaluated, computing the correlations and coefficients of determination. The evaluation was based on studies carried out by various authors and in different periods, namely Tundisi (1981), Trinidade (1988), Mateus and Tundisi (1988), UNEP-IETC (2001) and Rietzler (1995), Tundisi and Matmusura –Tundisi (1990) and Oishi (1996) in the reservoirs of Barra Bonita and Carlos Botelho (Broa), respectively.

3. Results

The two studied reservoirs differed widely in morphometric characteristics and in residence times (Table 1)

<table>
<thead>
<tr>
<th>Reservoirs</th>
<th>BROA</th>
<th>BARRA BONITA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basin</td>
<td>Itaqueri Basin</td>
<td>Middle Tiete Basin</td>
</tr>
<tr>
<td>Latitude</td>
<td>22° 29’ S</td>
<td>22° 31’ S</td>
</tr>
<tr>
<td>Longitude</td>
<td>48° 10’ W</td>
<td>48° 33’ W</td>
</tr>
<tr>
<td>Accumulated volume</td>
<td>22x10^6 m(^3)</td>
<td>3600x10^6 m(^3)</td>
</tr>
<tr>
<td>Area of reservoir</td>
<td>6.8 km(^2)</td>
<td>340 km(^2)</td>
</tr>
<tr>
<td>Area of basin</td>
<td>227.7 km(^2)</td>
<td>32.330 km(^2)</td>
</tr>
<tr>
<td>Maximal depth</td>
<td>12.0 m</td>
<td>25.0 m</td>
</tr>
<tr>
<td>Maximal length</td>
<td>7.5 km</td>
<td>50 km</td>
</tr>
<tr>
<td>Flow rate (min and max)</td>
<td>3.031 – 4.49 m(^3)/s</td>
<td>200- 950 m(^3)/s</td>
</tr>
<tr>
<td>Retention time</td>
<td>20 to 40 days</td>
<td>30 to 180 days</td>
</tr>
<tr>
<td>Vegetation</td>
<td>Cerrado, gallery forest and reforestation</td>
<td>Atlantic Forest</td>
</tr>
<tr>
<td>Perimeter</td>
<td>21 km</td>
<td>525 km</td>
</tr>
<tr>
<td>Average depth</td>
<td>3.2 m</td>
<td>11.53</td>
</tr>
<tr>
<td>Wind speed</td>
<td>3.8 to 8.1 m/s</td>
<td>5 to 7 m/s</td>
</tr>
<tr>
<td>Involvement factor</td>
<td>F = Ab + Ar/V</td>
<td>10.65</td>
</tr>
</tbody>
</table>

Accumulated wind speed values in the Barra Bonita reservoir were obtained in the period of January to December from Tundisi and Matmusura –Tundisi (1990)(see Figure 1).
In relation to the Broa (Carlos Botelho) reservoir, the data were obtained in the period of 1992 – 1998 from the climatology station of Centro de Recursos Hídricos e Ecologia Aplicada close to the Broa reservoir (Figure 2).

Figure 1. Wind speed in the Barra Bonita reservoir in the period of september 1983 to august 1984 from Tundisi and Matsumura –Tundisi (1990).

Figure 2: Daily wind speed in the reservoir of the Carlos Botelho dam over months of the year, in the period of July to December with strong winds and of January to June with weak winds, during 1992 to 1998.

The data of nutrients in the Broa reservoir were obtained from reports published by Tundisi (1981), Trindade (1988), Mateus and Tundisi (1988). See Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Nitrate (mg/L)</th>
<th>Total Phosphorus(mg/L)</th>
<th>Reactive Silicate (mg/L)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer 1971</td>
<td>1.258</td>
<td>15.21</td>
<td>1.66</td>
<td>Tundisi (1981)</td>
</tr>
<tr>
<td>Winter 1971</td>
<td>0.446</td>
<td>12.27</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>Winter 1980</td>
<td>0.356</td>
<td>2.43</td>
<td>18.88</td>
<td></td>
</tr>
<tr>
<td>Summer 1985</td>
<td>20.21</td>
<td>16.46</td>
<td>2.73</td>
<td>Mateus&amp;Tundisi,1988</td>
</tr>
<tr>
<td>Winter 1985</td>
<td>24.66</td>
<td>9.08</td>
<td>3.55</td>
<td></td>
</tr>
</tbody>
</table>

The data on nutrients in Broa reservoir show a change in 1971. Phosphate predominated in summer and winter according to Tundisi (1981), while in 1985, nitrates predominated in the two periods (Mateus and Tundisi, 1988). Trindade (1980) indicated a predominance of silicate in winter 1980. In 1991, there was a change with a higher level of phosphate in summer and one of silicate in winter, as reported by Oishi (1996)(see Figure 3).
Figure 3: Variation in the nutrients nitrate, phosphate and silicate in the periods of summer and winter in 1971, 1980, 1985 in the Broa reservoir.

In the Barra Bonita reservoir, the Primary production, chlorophyll, conductivity and ammonium measurements were taken by Tundisi and Matmusura – Tundisi (1990) are shown in Figure 4.
Figure 4: Fluctuations of chemical variables in the period of winter (May – September) and summer (November- April), in the reservoir of the Barra Bonita power plant. Study performed in 1983-1984 by Tundisi and Matsumura – Tundisi 1990.

Water temperature profiles Wedderburn and Richardson values in the Broa and Barra Bonita reservoirs indicated that the two reservoirs were stratified presenting the epilimnion, metalimnion and hypolimnion layers at different depths (Figure 5).

Figure 5: Temperature profiles with Wedderburn and Richardson values in the Carlos Botelho (Broa) reservoir in 1996. Data of biological variables from the Barra Bonita reservoir are presented in Figures 6, 7 and 8. The abundance of phytoplankton classes in the Barra Bonita reservoir is from September 1983 to August 1984, according to Tundisi-Matmusura- Tundisi, 1990). The data show occurrences in summer for Bacillariophyta, Chlorophyta in summer and autumn, Cyanophyta in autumn and spring, and Phytoflagellates throughout the whole year.
The density of secondary production characterised by the zooplankton *Thermocyclops* and *Mesocyclops* (Figure 8) showed greater occurrence in winter and spring.

**Figure 6:** Fluctuation of phytoplankton group in Barra Bonita reservoir

**Figure 7:** Percentage participation of phytoplankton group
Figure: 8: Distribution of zooplankton in the reservoir of the Barra Bonita power plant, in the period of January to December of 1992-1993, showing the variation in distribution of zooplanktonic organisms over the year, with greatest abundance in the periods of winter from May to October and low density in summer from November to April, according to the data of Rietzler (1995) corrected by Matsumura-Tundisi and Silva (2002).

In relation to secondary production, Figure 8 shows that there was greater occurrence of *Mesocyclops kieferi* in spring while *Thermocyclops decipiens* occurred in winter.
Figure 9: Values of non-dimensional Wedderburn, Richardson and lake numbers in two reservoirs

The results for the Wedderburn (W) number, Richardson (Ri) and Lake number (LN) of the two reservoirs shown in Figure 9 reveal the existence of 2 periods in both reservoirs, where the highest W values were in December in Barra Bonita (W = 50), and in March in Broa (W = 30). In relation to LN, the highest values were found in Broa in August (LN = 10^{-3}) and in Barra Bonita in July (LN = 600), while for Ri, the highest values obtained in Broa were in the month of March (Ri = 500), and in Barra Bonita in December (2.5 \times 10^{-7}). Comparing the graphs, it is seen that in Barra Bonita and Broa, the mean values were 9.81 and 2.88, and that the coefficients of variation were 1.24 and 3.15, respectively. The two reservoirs had the following mean values of Ri: Barra Bonita (6.2 \times 10^{-5}) and Broa (2.88); the coefficients of variation were of the same order, 1.02 and 2.51. The calculation of Lake number indicated that mean values were 9.92 \times 10^{-4} and 2.95 \times 10^{-4} in the Barra Bonita and Broa reservoirs, respectively. The analysis of coefficient of determination between the Wedderburn and Richardson non-dimensional numbers and the levels of nutrients in the Broa reservoir indicated some positive correlations, but only significant with Bacillariophyceae observed in Figure 10.

The examination of the figures indicates a positive relation between Lake number and biological parameters, but no significant relation with the nutrients.

4. Discussion

The reservoirs of Barra Bonita and Broa located in the neotropical region, according to Figure 2, are subject to strong winds from July to December and weak winds from January to June. These phenomena affect the structure of the reservoirs going through a succession of phases of circulation and stratification (Figure 5). Barra Bonita reservoir, as far as Tundisi et al. (2008) are concerned, is polymictic with fluctuation caused by rainwater and wind.
The external forces that influence the processes of horizontal and vertical mixture are wind, intrusion of water from tributaries, and low retention time in summer (December to March). Figure 9 shows the values of Wedderburn, Richardson and lake numbers. These figures reveal that the Richardson (Ri) and Wedderburn (W) values appear when there is considerable variation in temperature and density on comparing the results with the values of the thermal profile. Their amplitudes (width) depend on the thickness of the thermocline, that is, the larger the thermocline, the greater the values. These observations are confirmed by the calculation of W. In Barra Bonita, during spring and summer, when there is thermal stability of the water column, there are high values of the Wedderburn number (58) (Tundisi et al., 2002), values similar to those of this study. The Wedderburn number showed a value 3 < W < 10, on the days March 7, 1996 (6.75) and March 1996 (8.45), with the wind speed of 0.95 m.s\(^{-1}\) and 1.00 m.s\(^{-1}\) where daily stratification was weak (Figure 5). The thermocline of the surface formed by cooling of the surface water is moved by the effect of the wind, producing superficial turbulences, but the stratified structure of the reservoir is maintained. In September 1983, a speed of 4.05 m.s\(^{-1}\) resulted in W = 1.55 < 3, indicating strong winds. Important horizontal mixing shifted the thermocline deeper, disrupting the stratified structure of the reservoir. On the other days, W was greater than 10. Weak winds resulted in a strong stratification of the reservoir. With regard to the Wedderburn number (W), Figure 9 showed that in Broa, besides in March, W < 3 characterised mixing by strong winds or by mixing of density. The value of W > 3 indicated weak winds with the thermocline close to the surface. Horizontal mixing is important daily, where it cannot affect the reservoir in its totality, and as a final result, a stratified structure is formed. In relation to Barra Bonita, only in the months of February, March, May and September were the values between 3 and 10, showing a daily stratification marked by a shift in the thermocline due to the effect of strong winds. In May, the value W = 0.81 < 3 indicated strong winds. Applied to the reservoirs, the Wedderburn number showed that Barra Bonita was more subject to wind action. In the months of February, September and December, this parameter assumed values of 10.65, 12.92 and 46.85, respectively (Salencon and Thebault, 1997). The structure of the reservoir was dominated by the action of strong winds in the months of January, March, April, May, July, August and November, a period in which the Wedderburn number was between 3 and 10, showing weak stratification. The partial vertical circulations were observed in the months of June and October where the Wedderburn number was 2.7andzero (0), respectively, indicating strong winds or thermocline close to the surface. The structure of the reservoir on June 20th cannot be explained by this affirmation, because on that day, there was no record of wind by the method utilised. Thus, the value of W is linked more to the phenomenon of circulation independent of causative source of circulation.

In Broa, only in March, 1996, with the wind = 1.00 m.s\(^{-1}\) and 18 mm or rain, the Wedderburn number was > 10, i.e., 28.80. In the other months, the values were less than 3. Studies carried out in the Tucurui Reservoir, Pará state, indicated that Wedderburn numbers fell between 3 and 10 only in April and September, where they were 8.2 and 8.5, respectively, characterising weak stratification; in the month of May, the value was 0.81, i.e., less than 3 (0.81 < 3). Consequently, the thermocline was located close to the surface. In other periods of the year W >>10, because of weak winds and stratified structure (Nyamien & Tundisi, 2017). The dynamics of the mixing layer is determined by the magnitude of the Wedderburn number. In this case, turbulence in the layer is due to the force of the wind. The calculated lake number in all reservoirs for the whole year was less than 1, indicating a reservoir with stratified structure. LN = 0 occurred only in June in Barra Bonita, demonstrating a reservoir in circulation. The values found induce a periodic stratification to counteract weak winds. With respect to the Wedderburn numbers, the results were dominated by high values, over 10, confirming strong daily stratification and weak winds. In relation to Richardson (Ri), the values showed that the whole period (Ri < 0.25) confers to the reservoir little mixing of layers, and thus stable flows. Ri = 0 <0.25 characterised instability in virtue of microturbulence only in June in Barra Bonita. As a consequence, a turbulence of large whirlpools associated with unstable flow is created. The mixing layer is dominated by the turbulence produced on the surface and transported to the base of the mixing layer in which the thermocline disappears. The unidimensional processes are dominant. Because of the thermocline, the mixing layer moves under the effect of the wind, and the unidimensional structure is maintained. In this case, there are vertical circulations from the bottom to the top. The upwelling and horizontal mixing turn out to be important as a result of the circulation of body of water under the effect of these forces. Convective circulation caused by the difference in temperature leads to the transfer of heat from the water surface to the atmosphere because of a reduction in temperature and increase in water density. The inversion of density is then reversed with the downward shift of the denser superficial layer. The Richardson number (Ri) in Barra Bonita showed for the whole year a value of Ri < 0.25, confirming the stratified structure subject to cross flow, characterising an unstable flow due to microturbulences of large whirlpools associated with unstable flow. In the Broa Reservoir, on the contrary, all values are less than 0.25, characterising an unstable flow with little mixing between the layers. In relation to the effect on primary production in the Barra Bonita Reservoir (Figure 10), after the evaluation, only Lake number presented a positive relation with Chlorophyta, Cyanophyta and Phytolflagellate and a negative relation with Bacillariophyta, a response which was similar to the findings of Tundisi et al. (2002) with Aulacoseira sp. in the Carlos Botelho Reservoir.
With regard to the nutrients nitrate, total phosphate and total dissolved phosphate, a correlation was not found due to insufficient data in the period. Other studies must be realized to see this relation.

5. Conclusion

The Barra Bonita and Broa reservoirs are sensitive to wind action. They have a stratified structure throughout the year due to weak winds, where there is an influence on the development of some families of phytoplankton. The effect of non-dimensional numbers is linked to circulation movements independent of the source responsible for these movements. With respect to the availability of nutrients, this effect must be more evaluated and associated with other factors.

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6. References


7. Legends of Figures

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Figure 10: The coefficients of determination between nutrients and non-dimensional numbers Lake Number.