Performance Evaluation of Minimum Power Assignments Algorithms for Wireless Ad Hoc Networks

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Abstract

Energy consumption is a crucial issue in wireless ad hoc networks. Since many hosts in wireless ad hoc networks are usually powered by batteries, this makes the lifetime of a network to depend on the battery life of each individual host present. Minimizing the total energy consumption of each host is one of the major ways of improving the energy efficiency of a network. Therefore, a suitable minimum transmission power is assigned to wireless ad hoc communication station, such that single transmission of a node can be received by any node within its coverage area without affecting the performance of the system, thus reducing the total power consumed. This paper focuses on the performance evaluation of minimum power assignment Algorithms in wireless ad hoc network modeled in Matlab environment. The total power consumed by all the nodes in the network was minimized using the two min- total power assignment algorithms: greedy and Yao; and also the Minimum Spanning Tree (MST) based method. The performances of the three algorithms were evaluated and compared so as to determine the algorithm with the best reducing power. The simulation results show that the spanning ratios of communication graphs induced by using greedy- based and Yao-based power assignment methods are not as large as the MST-based method. Thus, the larger spanning ratio value of MST based method makes it to be less efficient in using it to solve minimum power assignments problem. It was observed that the Greedy and Yao based power assignment methods are suitable for power assignment tasks for wireless ad hoc networks due to their ability to have a bounded spanning ratio.

Keywords: Ad hoc wireless networks, Spanner, undirected graph, Spanning Ratio, Tree, Shortest spanning tree.

1.0 Introduction

Recently, the use of ad hoc wireless networks has received significant attention due to their potential applications in battlefield, emergency disaster relief and other applications. Unlike wired networks or cellular networks, no wired backbone infrastructure is installed in ad hoc wireless networks (Pen-Jun et al, 2005); and a communication session is achieved either through a single-hop transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise. All nodes transmit and receive signals through the use of omni-directional antennas which are attractive in their broadcast nature. A single transmission by a node can be received by many nodes within its transmission area.

This feature is extremely useful for multicasting/broadcasting communications. For the purpose of energy conservation, each node can dynamically adjust its transmitting power based on the distance to the receiving node and the background noise. According to (Rappaport, 1996), the signal power falls as $1/r^{\kappa}$, where r is the distance from the transmitter antenna and κ is a real *constant* between 2 and 4 depending on the wireless environment; Assuming all receivers have the same power threshold for signal detection, which is typically normalized to one. With these assumptions, the power required to support a link between two nodes separated by a distance r is r^{κ}. It was observed that relaying a signal between two nodes may result in lower total transmission power than communicating over a large distance due to the nonlinear power attenuation (Pen-Jun et al, 2005).

By power assignment for wireless ad hoc networks, it means to assign power for each wireless node such that the length of the shortest path in the induced communication graph is at most constant times of the length of the shortest path in the original communication graph when all nodes have the maximum power (Yu-Wang and Xiang-Yang, 2004) and (Lefteris et al, 2000). The need to evaluate the performance of minimum power assignments algorithms for wireless ad hoc network is very important because in literature, most of the existing works were based on designing different algorithm models that can reduce the energy consumption of a network.

2. 0 Related Work

The minimum energy connectivity problem was first studied by (Chen and Huang, 1989), in which the induced communication graph is strongly connected while the total power assignment was minimized. Recently, this problem has been heavily studied and many approximation algorithms have been proposed with the network modeled by using symmetric links or asymmetric links (Andrea et al, 2000), (Blough et al, 2000), and (Joseph et al, 2002). Several authors; (Pen-Jun et al, 2005), (Joseph et al, 2002), (Gruia et al, 2003) and (Mohammad et al, 2003); considered the minimum total power assignment while the resulting network is *k*-strongly connected or *k*-connected. Solving this problem can improve the fault tolerance of the network. The minimum energy connectivity problem was considered while the induced communication graph has a diameter bounded by a constant *h* (Cagalj et al, 2002). Other relevant work in the area of power assignment (or called energy-efficiency) includes energy-efficient broadcasting and multicasting in wireless networks. The problem, given a source node *s*, is to find a minimum power assignment such that the induced communication graph contains a spanning tree rooted at *s*. In (Chen and Huang, 1989), constructing a minimum-energy broadcast tree rooted at the source node was the focus. The nodes belonging to a broadcast tree were divided into two categories: relay nodes and leaf nodes. The relay nodes are those that relay data by transmitting it to other nodes (relaying or leaf), while leaf nodes only receive data. Each node can transmit at different power levels and thus reach a different number of neighboring nodes. When given the source node r, a set consisting of pairs of relaying nodes and their respective transmission levels was found so that all nodes in the network receive a message sent by r, and the total energy expenditure for the task was minimized. This broadcasting problem was called the minimum-energy broadcast problem.

3.0 System Model

Let power $\mathrm{P_{vu}}$ needed to support the communication between two nodes u and v which is a monotone increasing function of the Euclidean distance $\|uv\|$, (Yu-Wang and Xiang-Yang, 2004), and if

$$\mathrm{P_{vu}} > \mathrm{P_{xy}} \tag{1}$$

Then

$$\|uv\| > \|xy\| \tag{2}$$

Also if

$$\mathrm{P_{vu}} = \mathrm{P_{xy}} \tag{3}$$

Then

$$\|uv\| = \|xy\| \tag{4}$$

So for node V of the network, the power required to reach another node U is given by

$$\mathrm{P_{uv}} = (\mathrm{d_{vu}})^{\mathrm{k}} \tag{5}$$

Where $\mathrm{d_{vu}}$is the Euclidean distance between node u and v and $2 \le \mathrm{k} \le 4$ is a channel loss exponent. However, if power transmitted from node v cannot reach node u even when it transmiting at its maximum power, i. e

$$(\mathrm{d_{vu}})^{\mathrm{k}} > \mathrm{P_{vu(max)}} \tag{6}$$

Then $\mathrm{P_{uv}}$ is being redefined as $+\infty$

4.0 Building the Communication Graphs Using Matlab Script

The following algorithms were used in developing the communication graphs and evaluating their performances. Generally, nodes in an ad-hoc network are mobile but in this paper it is assumed that the nodes are relatively static.

4.1 UDG - Algorithm 1: Min-Max Power Assignment

1. Inputs: V which is a set of n wireless node and ε_{max} the maximum node power set at 3
2. Outputs: node powers
3. BEGIN
4. First build the Unit disk graph UDG
5. IF $\omega_{uv} \leq \varepsilon_{max,}$ there is an edge; END IF
6. IF $\omega_{uv} \geq \varepsilon_{max}$, threre is no edge; END IF
7. Sort the weights of all edges $uv \in UDG$
8. Get all possible node powers $w_1, w_2 \ldots, w_m$, where $w_1 < w_2 < \ldots < w_m \leq \varepsilon_{max}$ and $m \leq n^2$ is at most the number of links in UDG.

END

4.2 Algorithm *2:* Greedy Min-Total Power Assignment

1. Inputs: V and ε_{max}
2. Ouputs: Induced power assignment P_G
3. BEGIN
4. Building UDG: Using *V* and ε_{max}, we first build the unit disk graph *UDG*.
5. Sorting UDG edges: Sorting edges in UDG according their weights get $e_1, e_2, \ldots, e_m$, where $we_1 < we_2 < \ldots < we_m \leq \varepsilon_{max}$
6. Greedy method: Initialize *G* to be an empty graph. Following the increasing order, add an edge ei = *uv* to *G* if and only if no path in *G* (already added edges) with total power no more than $t_0 . \|uv\|^2$.
7. Power assignment: Extract the induced power assignment P_G, where $P_G(u) = \max\{v/uv\epsilon G\} w_{uv.}$

END

4.3 Algorithm *3:* Yao-Based Min-Total Power Assignment

1. Inputs: V and ε_{max}
2. Ouputs: Induced power assignment P_G
3. BEGIN
4. Building UDG: Using *V* and ε_{max}, we first build the unit disk graph *UDG*.
5. Building Yao graph: Set $k \geq \pi / \arcsin \frac{\sqrt{1-\frac{1}{t_0}}}{2}$, apply $Y\,G_k$ on UDG. For each node *u*, assume that it has d_u edges $uv_1, uv_2, \ldots, uvd_u$ in UDG. Then for each edge uv_i, we can assign a cone partition C_i (one of the cones started at link uv_i). We test Yao structure of *u* for all the d_u cone partitions C_i, and select the one whose maximum chosen link incident is the smallest. Then the union of the Yao structures of all nodes forms a graph *G*.
6. Power assignment: Extract the induced power assignment P_G, where $P_G(u) = \max\{v/uv\epsilon G\} w_{uv.}$

END

4.4 Algorithm *4:* Minimum Spanning Tree Method Using Kruskal Algorithm

Given a connected graph G = *(V, E)* with edges *(i.j)* having length *lij* > O. the algorithm determines a shortest spanning tree Tin G.

1. Inputs: Edges *(i, j)* of G and their lengths *lij*
2. Outputs: Shortest spanning tree *T* in G
3. Order the edges of G in ascending order of length.
4. Choose them in this order as edges of *T*, rejecting an edge only if it forms a cycle with edges already chosen.
   If n - I edges have been chosen. then
   Output *T* (= the set of edges chosen). Stop

END

❖ Definition of Terms

Tree T: is a graph that is connected and has no cycles. "Connected" means that there is a path from a vertex in T to any other vertex

Spanning tree T: in a given connected graph G = (V ∗ E)is a tree containing all the n vertices of G. and such tree has n-1 edges.

Shortest spanning tree T: in a connected graph G (whose edge (i, j) have length l_{ij}>0) is a spanning tree for which Σl_{ij} (sum over all edges of T) is minimum compared to $\Sigma l_{ij}$for any spanning tree of G.

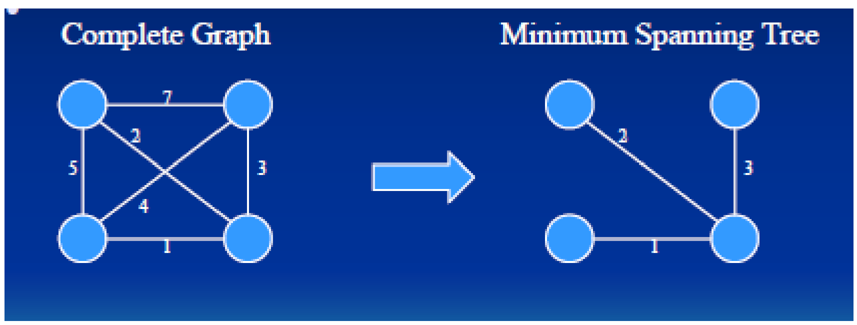
Spanner: means that the length of the shortest path in the induced communication graph is at most constant times of the length of the shortest path in the original communication graph.

Spanning Ratio: If a subgraph *H* = (*V*, *E* ′) is a *t-spanner* of *G* = (*V*, *E*) if for every u, v ϵ V, the length (or weight) of the shortest path between them in *H* is at most *t* times of the length of the shortest path between them in *G*. The value of *t* is called the *stretch factor* or *spanning ratio*.

UDG: undirected graph

Example of Spanning Tree is shown in Figure 1, the minimum spanning tree for a given graph as the spanning tree of minimum cost for that graph was described in (Hardik and Soujanya, 2001)

Figure 1: Minimum Spanning Tree Graphs



Source: (Blough et al, 2000)

5.0 Communication Graph Model

The communication graph is achieved by considering a set $V = \{ v_1, v_2 \ldots, v_n\}$ of *n* wireless nodes distributed in a two dimensional plane with edges uv, the weight function ω_{uv} is the power needed to support the communication link between the two nodes. Specifically, each node *u* has a maximum transmission power ε_{max} and it is assumed that it can adjust its power to be exactly ω_{uv} to support the communication to another node *v*. Consequently, if all wireless nodes transmit in their maximum power, they define a wireless network that has a link *uv* iff $\omega_{uv} \leq \varepsilon_{max}$ [3]. This forms the original communication graph also known as the unit disk graph (UDG). The induced communication graph is generated by applying the two min-total power assignment methods (i.e. Greedy and Yao) and also the MST based method to assign power for each node. Then the total power assignments were compared. Hence, Figure 2 shows a set of 100 wireless stations (nodes) in a plane and line connecting the nodes with different range of transmission, where any station transmits directly to all other station and indirectly to all other station via a relay station.

6.0 Results and Discussions

6.1Graphical Models

The simulation that show the evaluation of the performance of the two min- total power assignment algorithms (greedy and Yao; and also the MST based methods) was carried out in MATLAB software environment. MATLAB codes were written for the implementation of the models and simulated to generate results in form of graphs for visualization. The performances of the three algorithms were compared and the algorithm with the best reducing power was determined.

After building the original communication graph as shown in Figure 2, the induced communication graph was built using the three algorithm methods. Figure 3 shows the induced communication graph after applying the Greedy method to the original communication graph). Comparing this method with the Unit disk graph (UDG) in Figure 2, it can be seen despite the fact that the number of nodes in the two graphs remain the same, the number of links formed in the graph using Greedy method has reduced drastically also with lesser closed loops whereas the graph is still connected. This reduction in the number of links will result in a minimized total power consumed thereby increasing the energy efficiency of the network. Also, Figure 4 shows the induced communication graph after applying the Yao method to the original communication graph. It is clear from the graph that Yao method keeps more links than Greedy method. It can be seen that despite the fact that Yao method keeps more links, the links has reduced to an extent and also the number of links formed in the induced communication graph of the Yao method is not as dense as that of the UDG in Figure 2. Also the closed loops formed by the method were not greatly reduced like that of the greedy method. The ability of the Yao method to keep more links makes it to consume more power. Consequently, Figure 5 shows the induced communication graph after applying the MST (Minimum Spanning Tree) method to the original communication graph). The method keeps lesser links than all other methods and it does not form any closed loop like other methods. This method has been able to greatly reduce the number of links formed by the network without the formation of any closed loops with the network been perfectly connected, this leads to a greater reduction in the total power consumed.

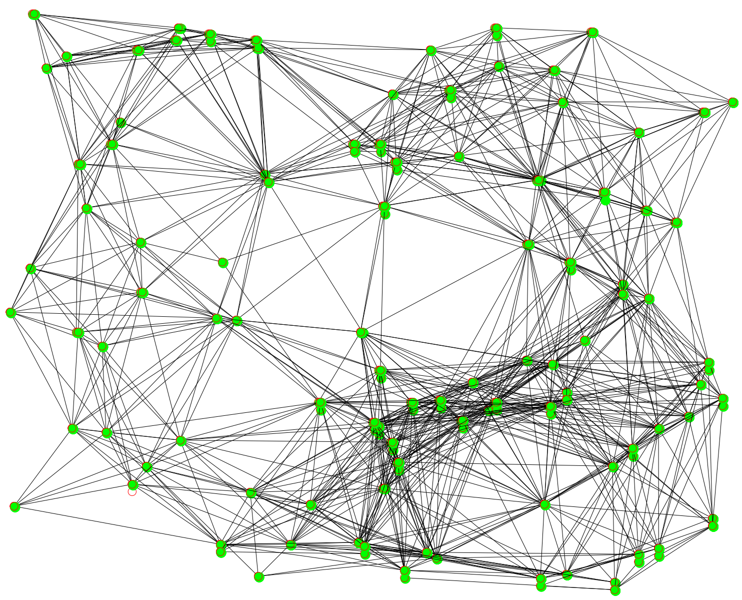


Figure 2: Original communication graph (UDG) of 100 set of network node

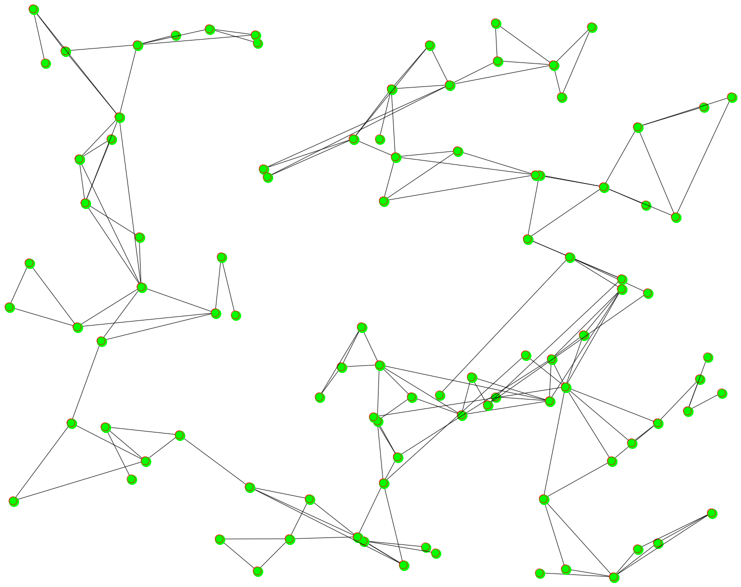


Figure 3: Induced communication graph using Greedy Method

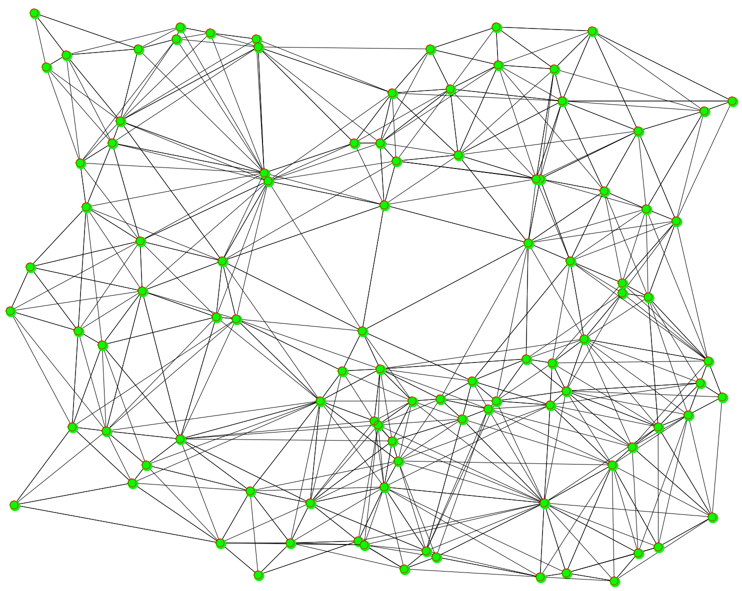


Figure 4: Induced Communication Graph Using Yao Method

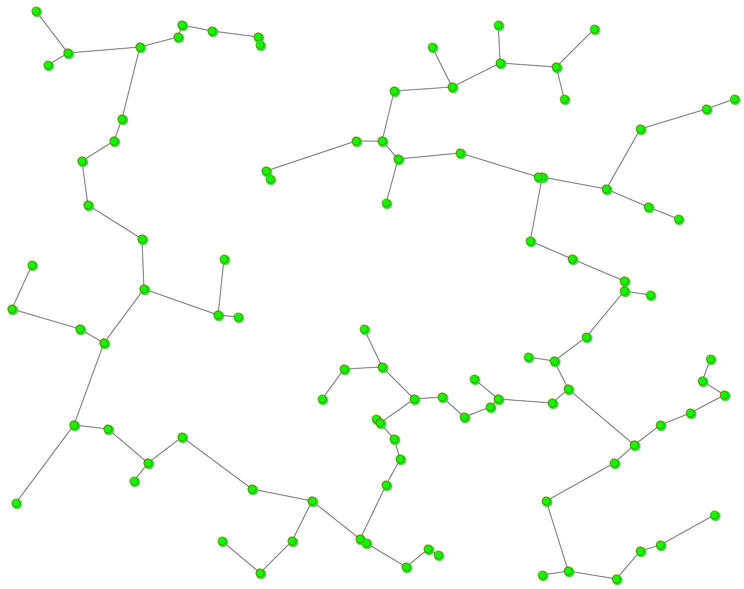


Figure 5: Induced Communication Graph Using MST Method

6.2: Numerical Results

Table 1 compares the performances of the three methods with their different total assigned power and spanning ratios. All the results in Table 1 were all gotten from different iterations using MATLAB Software Package. The results show that the spanning ratios of communication graphs induced by using greedy-based and Yao-based power assignment methods were not as large as the MST-based method. The larger spanning ratio value of MST based method makes it to be less efficient in using it to solve minimum power assignments. It can also be seen from the result in Table 1 that Yao-based method spends more power due to its ability to keep more links, more so it is also easy to perform and can be run locally. However, Greedy and Yao based power assignment methods are suitable for power assignment tasks for wireless ad hoc networks due to their ability to have a bounded spanning ratio.

Table 1: Total Assigned Power and Spanning Ratios of Graphs Induced by Different Min Total Power Assignment Methods

	MST	GREEDY	YAO
Avg Total-Power (P (G)	84.08	248.71	429.15
Avg Total-Power (P (UDG)	491.70	492.50	491.60
Avg (P(G) / P(MST)	1.000	2.957	5.103
Max (P(G) / P(MST)	1.000	2.202	5.327
Avg Spanning Ratio	2.306	1.080	1.040
Max Spanning Ratio	15.888	1.682	1.456

Finally, the numbers of nodes in the region were fixed at 100 and the transmission range of each node was varied from 3.0 to 5.0 and the performances of all the structures were plotted. From Figure 6, it can be seen that at transmission range of 3; for the MST method, the total node power was around 80 whereas at the range of 5 the total node power increased a bit but not feasible. For the Greedy method; at the transmission range of 3.5, the total node power was around 170 also at the range of 5 the total node power increased drastically to 440. This means that its total node power increases as the transmission range increases. Also, for the Yao method its total node power also increases a bit as the transmission range was varied. The total node power for MST was greatly reduced due to its lesser links and not forming of any closed loop, followed by the Greedy method and the Yao method that keeps more links. Also, varying the transmission range from 3 to 5 and the average total node power of each of the three methods were compared to the UDG (Undirected Graph) as shown in Figure 7.

MST method showed a greater reduction in node power as the transmission range was varied from 3 to 5 while the total node power compared to UDG in the GREEDY method increases as the transmission range was increased. Also for the YAO method, its total node power compared to UDG decreased drastically as the transmission range was increased from 3 to 5.

Figure 8 shows the performance of the average total node power of the three methods when compared with one of the methods which is MST while the transmission range was varied from 3 to 5. MST method was on the same level without an increase in its node power since it was been compared with it. The total node power compared to MST increases drastically when the transmission range was varied in the GREEDY method whereas the total node power compared to MST for the YAO method only increased a bit.

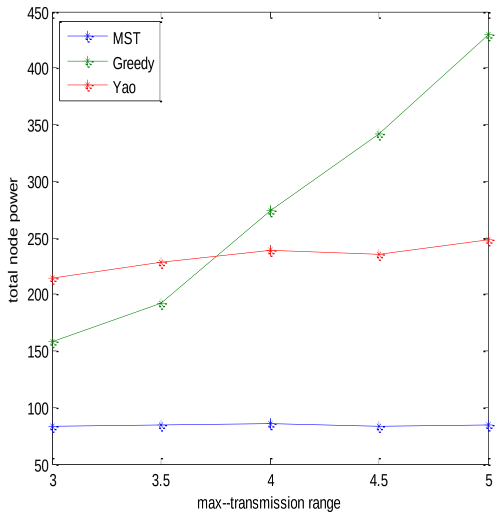


Figure 6: Total Node Power

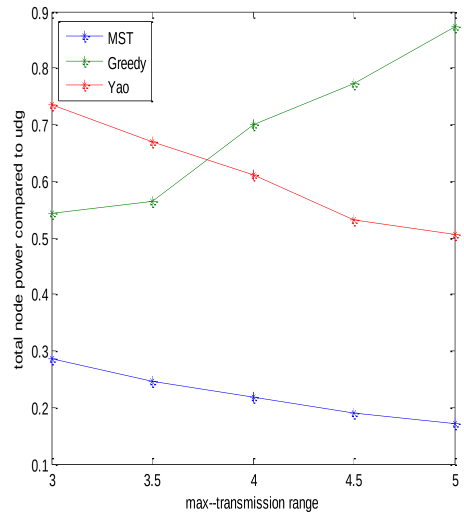


Figure 7: Average-total-node-power-ratio-with- UDG

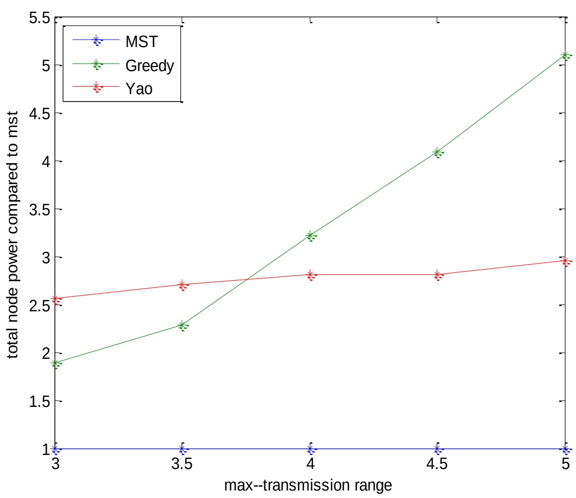


Figure 8: Average-total-node-power ratios-with- MST

Figure 9 depicts the performance of the maximum node power of the three methods when compared with one of the methods which is MST while the transmission range was varied from 3 to 5. MST method was on the same level without an increase in its node power. Greedy method still showed an increase in maximum total node power when compared with Yao method. When the transmission range was varied from 3 to 5 and the average spanning ratios of each of the three methods were plotted as shown in Figure 10. MST method showed an increase in its spanning ratio followed by Greedy and Yao method.

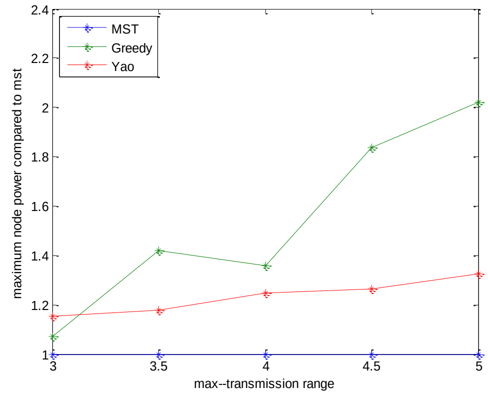


Figure 9: Maximum-total-node-power ratios-with-MST

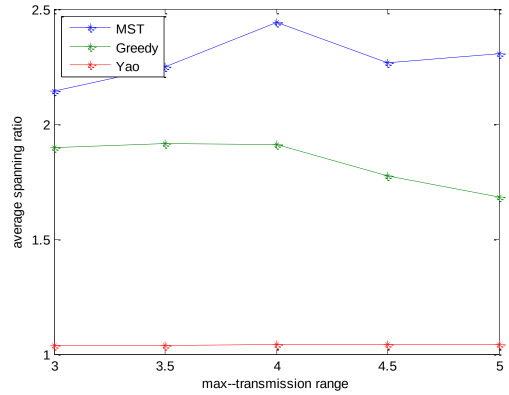


Figure 10: Average-spanning ratio

7.0 Conclusion

The results in this paper showed that the transmission powers of the induced communication graphs of the three algorithms were greatly minimized after the simulation process when compared to the original communication graph. It can also be observed that the spanning ratios of communication graphs induced by using greedy based and Yao-based power assignment methods are not as large as the MST-based method. The larger spanning ratio of MST based method makes it to be less efficient in using it to solve minimum power assignments. Both the Greedy and Yao method have a bounded spanning ratio and the methods were able to reduce the transmission power when compared to the total power of the original communication graph, however minimum energy was used in the network and the problem of power consumption by each node was greatly reduced to minimal and this will help to prolong the network life time and also save cost of transmission.

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