

Estimation of Failure Probability in Corroded Oil Pipelines through Monte Carlo Simulation Method Applying the Bootstrap Technique

Isabella Andreczewski Chaves

FAE-Centro Universitário/PPGMNE-UFPR
Universidade Federal do Paraná
Brazil

Roberto Dalledone Machado

PPGEM-PUCPR/PPGMNE-UFPR
Universidade Federal do Paraná
Brazil

Anselmo Chaves Neto

Statistics Department/PPGMNE-UFPR
Universidade Federal do Paraná
Curitiba, PR, Brazil

Abstract

This paper discusses the method of structural reliability analysis based on Monte Carlo simulation by applying the Bootstrap technique. The application of Monte Carlo method requires sample generation of random variables corresponding to strength characteristics and load of structural system for calculating the limit state equation. In this particular case, the probability distributions of some random variables involved are unknown. Therefore, the generation of random samples of distributions, in which Monte Carlo method is based on becomes infeasible. Thus, it was applied the Bootstrap technique in order to estimate the real sampling distribution of the statistical failure probability. By the Bootstrap distribution, it was estimated the failure probability by point and interval. The study has been applied in two real runs of equipment known as PIG Palito used in pipeline transportation of oil, gas and derivatives. It was obtained as a result, the failure probability pf , in each run as well as the corresponding confidence interval percentile Bootstrap.

Keywords: Structural Reliability, Monte Carlo Method, Bootstrap, Failure Probability, Confidence Interval Bootstrap

1. Introduction

The high technology required for extracting oil from deep-water rig in Brazil is largely dominated today and most of its oil production comes from fields located in this region. After the extraction of oil and gas, products are transported to the refinery oil plants and this task is performed mainly through pipelines. This operation is exposed to accidents related to landslides, rock fall, vandalism, interference of others and corrosion, among other things. Accidents can cause leaks as well as environment and socio-economic damages. Corrosion is a major cause of failures upon the pipeline network. Therefore, the transport of oil and gas through pipelines requires continuous monitoring and it can be done by means of instrumented inspections. Most existing monitoring methods do not consider the uncertainties associated with the variables involved in a problem of defect caused by corrosion in pipelines, such as the dimensions of the defect, geometrical characteristics of the pipeline and mechanical properties of the material. By not evaluating the uncertainty associated with variables, it is ignored the probability of a certain configuration of these variables bring on a fault condition in the duct. Structural reliability intends to determine the probability of a failure situation.

This paper aims to apply the Monte Carlo technique associated with the computationally intensive method known as Bootstrap to assess the failure probability, pf , and its confidence interval. The techniques were applied in pipelines subject to corrosion and the records obtained in two monitoring inspections which were run with an equipment called PIG Palito, were analyzed.

2. Introduction to Analysis of Structural Reliability

2.1 Basics Concepts

Structural Reliability is a set of techniques that allow the evaluation of the uncertainty present in a structural system. It is well-known that every Engineering project depends on technical characteristics that are random variables. Additionally, in the project design some decisions can be made on a condition of uncertainty. Wherefore, there is a chance that the project does not perform as expected or simply fail. Freudenthal (1947) states that to an engineering system that is subject to these conditions of uncertainty there is no guarantee of absolute safety then for this reason a risk is inevitable.

When it comes to a structure, safety depends on the strength of its components and the maximum load that the system is subject throughout its lifetime. Maximum load and strength are sometimes difficult to predict accurately, so that complete security can only be ensured in terms of probability. Hence, reliability can be defined as the probability measure of the performance guarantee, that is, the security performance can realistically be established only in terms of probability.

Probabilistic structural analysis is defined by Ditlevsen and Madsen (1996) as the art of formulating a mathematical model in which it can be proposed and also answered the following problem. "What is the probability that a structure behave in a specific manner, since some properties of materials or yet their geometrical dimensions are not completely known or even are from random nature?"

Structural reliability can be applied in calibration of the design codes, reanalysis of the existing structures, planning, inspection plans, and safety assessment of the new structural concepts as well as in the choice of design alternatives. The design norms often indicate load and strength partial factors that are defined based on the experience of professionals involved in the structural projects. Currently, there is a possibility from the use of structural reliability in calibration of safety factors with scientific basis, in addition to professional experience. This can be done from the definition of a level considered acceptable for the failure probability.

The structural reliability is considered a very important technique in engineering. However, it has to be warned that this depends on the quality of statistical data related to the problem and also of the precision of the mathematical modeling of the limit state function. Thus, to assess the reliability of system, initially it is necessary to define the random variables R = strength and S = load. Furthermore, the probability of the event ($R > S$) represented by $P(R > S)$ corresponds to the realistic reliability of the system. The probability of the complementary event $P(R < S)$ is the measure of non-reliability of the system.

Assuming that the probability distributions $F_R(r)$ or $f_R(r)$ and $F_S(s)$ or $f_S(s)$ and the limit state function $g(\underline{X})$ are known it is possible to calculate the failure probability, p_f , by:

$$p_f = \int_F f_{\underline{X}}(\underline{X}) d\underline{X} = P[g(\underline{X}) \leq 0.0] \quad (1)$$

With \underline{X} being the vector composed by related variables to strength and load, F the area of failure and $f_{\underline{X}}(\underline{X})$ the joint p.d.f. associated with the vector \underline{X} . Thus, the reliability of the system C can be defined as the complement of the failure probability, p_f , namely,

$$C = 1 - p_f \quad (2)$$

2.2 Analysis Methods of Structural Reliability

Real problems can present non-Gaussian variables and limit state function, $g(\underline{X})$, complex. Thereafter, the numerical calculation of the equation $p_f = \int_F f_{\underline{X}}(\underline{X}) d\underline{X}$ may be difficult to obtain. Thus, alternative methods are used to obtain the probability of failure, p_f . These methods are the analytic and Monte Carlo simulation. The analytic methods are the FOSM (First Order and Second Moment Reliability Method), the FORM (First Order Reliability Method) and SORM (Second Order Reliability Method). These methods are all well known in the literature and are well described in Ditlevsen and Madsen (1996) moreover Melchers (1987).

Monte Carlo simulation method is a process of repeated generation of pseudo deterministic solutions. Consequently, each solution corresponds to a set of values from the random variables generated. Therefore, in Monte Carlo simulation is processed the generation of random values of the specific probability distributions (Gaussian, lognormal, etc.).

The application of Monte Carlo simulation method for determining the probability of failure requires the generation of sample values of the structural variables involved in the limit state equation $g(\underline{X}) = g(X_1, X_2, \dots, X_p) = 0$, that is, observations of the random vector $\underline{X}' = [X_1, X_2, \dots, X_p]$ where the components are random characteristics of structural elements.

2.3 Bootstrap

Statistical Science is based on the recognition of uncertainty existence in almost all phenomena natural or caused by human activity. This science incorporates concepts such as probability models, likelihood, variance, standard errors and confidence intervals, among others that are used to build up techniques to assess the uncertainty of estimates of population parameters. Bootstrap concept is the following: “Bootstrap is a non-parametric statistical technique, computationally intensive, which allows assessment of the variability of a statistic $T_n(\underline{x})$ based on data from single original sample existing of size n , $\underline{x}' = [x_1, x_2, \dots, x_n]$. Choosing from the original sample \underline{x} , with replacement, B “samples Bootstrap” with same size n $\underline{x}_1^*, \underline{x}_2^*, \dots, \underline{x}_B^*$ and calculating the statistic in question for each of B Bootstrap samples”.

Bradley Efron in 1979, Efron (1979), introduced this technique to the scientific community. Since that time several theoretical and applied studies have been conducted in the area of statistics. Applications in other areas are also being developed, such as Picheny et al. (2010) who apply the method in the conservative estimation of reliability with limited samples. Pedroni et al. (2010) makes a comparison of Artificial Neural Networks with application of Bootstrap and Quadratic Response Surface for estimating the probability of functional failure and Kijewski et al. (2002) that implemented the Bootstrap on a reliability problem of a class of identification systems related to structural damping.

The technique application consist to do re-sampling with replacement from the original sample in order to obtain a very large number of Bootstrap samples, B . Together with these Bootstrap samples, which are known as pseudo data it is calculated the statistic of interest for each Bootstrap sample. The result is the set formed by the Bootstrap values of the statistic, namely $\{T_i^*(\underline{x}_i^*) \mid i = 1, 2, \dots, B\}$. This set of values is called Bootstrap distribution of the statistic $T_n(\underline{x})$.

It is the θ parameter and its estimator $T_n(\underline{x}, F)$, taking into account that the distribution of $T_n(\underline{x}, F)$ is unknown and assuming that $\underline{x}' = [x_1, x_2, \dots, x_n]$ is a random sample of size n available from random variable with unknown distribution function F . Thus, X_i is independent with distribution F and there is still the following information:

- The nonparametric maximum likelihood estimator of F is:

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^n I(X_i \leq x) \tag{3}$$

where $I(X_i \leq x)$ is the indicator function that is 1 if $X \leq 1$ and 0 in a contrary case. Then, this empirical distribution is constructed by placing probability mass, $1/n$, in each of the n sample points;

- It is taken a very large number, B , of the Bootstrap sample of equal size n of the distribution function, namely:

$$\{\underline{x}_i^* = [x_{i1}^*, x_{i2}^*, \dots, x_{in}^*], \quad i = 1, 2, \dots, B\} \tag{4}$$

- It is calculated the B Bootstrap statistics, that is, $\{T_i^*(\underline{x}_i^*) \mid i = 1, 2, \dots, B\}$ and it is obtained the Bootstrap estimation of the parameter θ and the variance of the θ estimator, $T_n(\underline{x}, F)$, using:

$$T_n^* = \frac{1}{B} \sum_{i=1}^B T(\underline{x}_i^*) \tag{5}$$

$$s^{*2} = \frac{1}{B-1} \sum_{i=1}^B [T_i^*(\underline{x}_i^*) - T_n^*]^2 \tag{6}$$

Then, $\{T_i^*(\underline{x}_i^*) \mid i = 1, 2, \dots, B\}$ is a simulation of the real statistical sample distribution of $T_n(\underline{x}, F)$ that produces a new parameter estimator and its variance. Along with the available set $\{T_i^*(\underline{x}_i^*) \mid i = 1, 2, \dots, B\}$ beyond of the Bootstrap standard error of $T_n(\underline{x}, F)$ it is possible to obtain a measure of bias using:

$$T_n(\underline{x}, F) - T_n^* \tag{7}$$

The Bootstrap distribution of $T_n(\underline{x}, F)$ is obtained using the method of Monte Carlo simulation with a number of replicates B . Bootstrap is by its nature, a product of the current stage of development of electronic computing data. In the 40s, 50s, 60s and even early 70s it would be unimaginable that if it was taken a sample of size $n = 30$ from a random variable, $[x_1, x_2, \dots, x_n]$, it would be possible to build up $B = 10,000$ Bootstrap random samples of size n , that is, $\underline{x}_1^*, \underline{x}_2^*, \dots, \underline{x}_B^*$, where $\underline{x}_i^* = [x_{i1}^*, x_{i2}^*, \dots, x_{in}^*] \mid i = 1, 2, \dots, B$, and finally if it was calculated B times the statistical $T_n(\underline{x})$ obtaining $T_i^*(\underline{x}_i^*) \mid i = 1, 2, \dots, B$. This set of estimates corresponds to the Bootstrap statistic distribution of $T_n(\underline{x})$.

The histogram of these $T_i^*(\underline{x}_i^*)$ values is a kind of picture of the true distribution of the statistic $T_n(\underline{x})$, which is unknown. An example is shown in Figure 1 for the statistical index of reliability $\hat{\beta}$ that has the punctual value $\hat{\beta} = 5.01$. For this index value corresponds a failure probability $p_f = 0.000003$, namely, three chances in 1,000,000. It was used $B = 10,000$ Bootstrap statistic values $\hat{\beta}^*$, that is, the set of values $\{\hat{\beta}_i^* \mid i = 1, 2, \dots, 10,000\}$, and based on this set it is possible to evaluate the variability of the statistical by the Bootstrap standard error, s^* , and build up a Bootstrap confidence interval for the true parameter β . The Figure 2 shows the construction algorithm of the Bootstrap distribution of the statistic $T_n(\underline{x}, F)$.

Figure 1 - Histogram of the reliability index $\hat{\beta}$

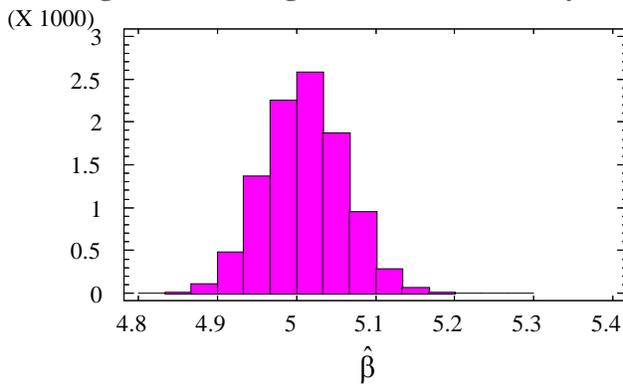
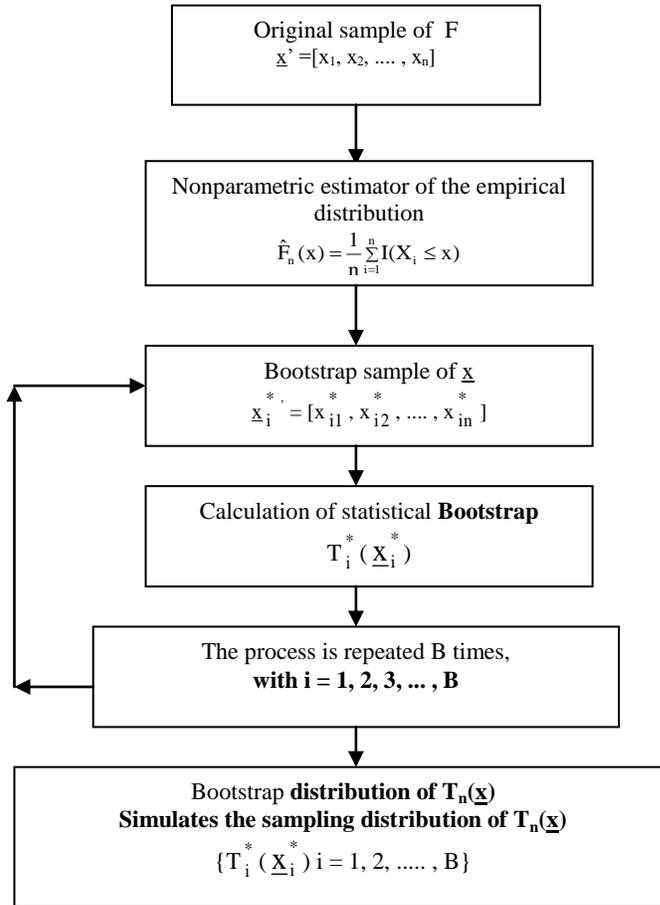


Figure 2: Flowchart of the Construction Algorithm of the Bootstrap Distribution of the Statistic $T_n(\underline{x}, F)$



As the Bootstrap was being developed, Bootstrap confidence intervals were created. The following describes these intervals.

A) Confidence Interval "Basic Bootstrap".

The original sample $\underline{x}' = [x_1 \ x_2 \ \dots \ x_n]$ provided the estimate t of the parameter θ which was obtained using the estimator $T_n(\underline{x})$ whose probability distribution is unknown. Then, considering the empirical distribution $\hat{F}_n(t)$ to $T_n(\underline{x})$ and applying the Bootstrap technique in the original sample B times the result is the set of Bootstrap estimates of θ , $[t_1^*, t_2^*, \dots, t_B^*]$. Sorting this set in an increasing way you have $[t_{(1)}^*, t_{(2)}^*, \dots, t_{(B)}^*]$ and taking into account the difference $(t_{(B+1)(\alpha/2)}^* - t)$ e $(t - t_{(B+1)(1-\alpha/2)}^*)$ the basic interval of level $1 - \alpha$ is given by:

$$[2t - t_{(B+1)(\alpha/2)}^* \ ; \ 2t - t_{(B+1)(1-\alpha/2)}^*] \tag{8}$$

where $t_{(B+1)(\alpha/2)}^*$ is the Bootstrap estimation of θ , such that $(B + 1)(\alpha/2)$ is an approximate integer order number which is greater than $(\alpha/2)\%$ from the terms of ordered set and similarly $(B + 1)(1 - \alpha/2)$ is another approximated integer which is greater than a $(1 - \alpha/2)\%$ of the ordered set of terms.

B) Confidence Interval "Studentized Bootstrap".

The original sample $\underline{x}' = [x_1 \ x_2 \ \dots \ x_n]$ provided the estimate of the parameter θ obtained using the estimator $T_n(\underline{x})$ whose probability distribution is unknown.

Therefore, considering the empirical distribution $\hat{F}_n(t)$ to $T_n(x)$ and applying the Bootstrap technique in the original sample B times it is obtained the set $[t_1^*, t_2^*, \dots, t_B^*]$ and the Bootstrap statistics:

$$\text{Bootstrap estimate of } \theta: \quad t_n^* = \frac{1}{B} \sum_{i=1}^B t_i^* \quad (9)$$

$$\text{Bootstrap standard error of } T_n(\underline{x}): \quad s^* = \sqrt{\frac{1}{B-1} \sum_{i=1}^B [t_i^*(\underline{x}_i) - t_n^*]^2} \quad (10)$$

$$\text{Therefore, with this greatness, the pivot estimate is built up } z = \frac{T - \theta}{\sigma} \text{ given by:} \quad (11)$$

$$z^* = \frac{t_n^* - t}{s^*} \quad (12)$$

In addition, the Bootstrap Studentized interval is:

$$[t - z_{(B+1)(\alpha/2)}^* s^* ; t - z_{(B+1)(1-\alpha/2)}^* s^*] \quad (13)$$

Being that the B estimates Bootstrap z_i^* $i = 1, 2, \dots, B$ are ordered.

C) Confidence Interval "Bootstrap Percentile"

The percentile interval is built considering the ordered values of Bootstrap estimates and taking to limits the values corresponding to positions $(B+1)(\alpha/2)$ and $(B+1)(1-\alpha/2)$ for a confidence level of $1 - \alpha$. Thus, there is:

$$[t_{(B+1)(\alpha/2)}^* ; t_{(B+1)(1-\alpha/2)}^*] \quad (14)$$

Simulations have shown that from the three intervals addressed, the more accurate is the "Studentized Bootstrap". For this paper it was applied the percentile method to construct the confidence interval for the reliability index β and the failure probability p_f .

2.4 Evaluation of Defects in Pipelines

Inspection and maintenance manual of pipelines, such as the PDAM manual (Pipeline Defect Assessment Manual - PDAM) recommends several methods for evaluation of pipelines in correspondence to various types of defects. The defects are considered along the pipeline and also about its circumference. Basically the methods work with the prediction of strength to a pipeline rupture defective and subject to internal pressure.

The inspection of pipelines is made by equipment called PIG - Pipeline Inspection Gauge. The work performed by this instrument is to check for irregularities (anomalies) in the pipeline tubing, such as parts of wrinkled internal surface, corrosion and also concave or convex ovalities. PIG also makes inspections at predetermined points with sensors of ultrasound or magnetic type. Generally the PIGs are nondestructive inspections of pipelines. PIG is a solid, rigid or flexible device that is inserted into the duct and moves with the flow of the transported fluid (oil, derivatives, or gas) to make the cleaning service and the inspection of potential discontinuities.

The basic technologies used to instrument the PIGs are MFL, or better, Magnetic Flux Leakage and another by Ultrasound. So there are MFL Magnetic Flux Leakage PIG and the Ultrasonic PIG. In the operation of an instrumented PIG with MFL technology, in a first phase a magnetization of the duct wall is done and in the next step is performed a measurement of the magnetic field upon the magnetized surface. In the case of irregularities on the duct wall, this is identified by the leakage of magnetic field captured by sensors. Leakage of magnetic field is seen as a loss of metal from the tube wall, that is, on the magnetized wall there is a corrosion causing a thinning of the pipe.

The ultrasound technology is based on the emission of high frequency waves against the wall of the duct and the capture echo caused by the wave. Instrumentation records the time of the wave departure and the returning of the echo.

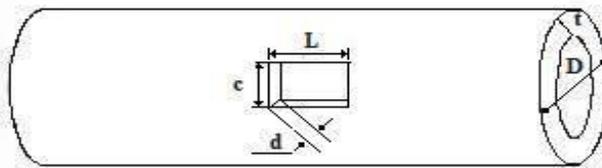
When the departure of the wave and the returning of the echo are in different times there has been evidence of irregularities in the pipe that might be corrosion. This technology is not subject to the thickness of the duct wall and can be placed in smaller equipment for inspection of pipelines with diameters from medium to small. This makes Ultrasonic PIG more advantageous over the MFL PIG. However, PIG Ultrasonic requires that the internal walls of the duct are cleaned thoroughly, that is, they have to be free from paraffin layer and other deposits derived from the transported fluid. This condition requires that the fluid flow has to be homogeneous in order to avoid deposits of various materials, particularly sand.

2.5 Strength Evaluation of Pipelines

Strength evaluation of pipelines is very well described by the following authors: Squarcio (2009), Lamb (2009), Vanhazebrouck (2008) and Cabral (2007). This assessment, when the ducts are subject to corrosion, is made with the application of continuum mechanics concepts and the incorporation of empirical results. Thus, we obtain analytical results that allow the estimation of burst pressure of pipes subject to corrosion defect.

Physical characteristics of a duct are shown in Figure 3 which is classical in the studies about pipelines strength carried out by, Cabral (2007), Choi (2003), Ahammed (1996, 1997 and 1998), Vanhazebrouck (2008) and Squarcio, (2009). In assessing the strength, the duct subject to corrosion has dimensions $L \times C \times t$.

Figure 3 – Pipeline's Measures Subjected to Corrosion



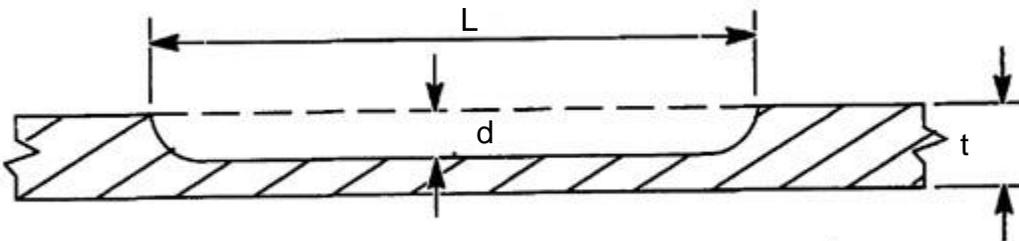
Source: Choi et al. (2003)

Figure 3 shows the following measures:

- D is the external diameter of pipeline;
- L is the length of corrosion in the pipeline;
- t is the thickness of the pipeline wall;
- d is the depth of corrosion;
- C is the width of corrosion.

The Figure 4, classic upon the cited works shows a cross section of the irregularity due to a corrosion of length L, width c and depth d.

Figure 4 –Cross Section of the Irregularity Corrosion



Source: Ahammed & Melchers (1996)

Figure 4 shows the following measures:

- L is the length of corrosion in the pipeline;
- t is the thickness of the pipeline wall;
- d is the depth of corrosion.

Thus, in corrosion occurrence, the wall of the duct is damaged by either breakage or material wear, furthermore by loss of the metal that composes it. Ahammed and Melchers (1996) also state that when the wall thickness is small compared to the diameter of the duct and the density of the transported fluid is low relative to its pressure, the relationship between the circumferential tension expected to the level of corrosion s_p , and the expected pressure of the fluid, P_p , is given by:

$$P_p = \frac{2s_p t}{D} \quad (15)$$

Then, performing the simple replacement s_p for its own expression namely:

$$s_p = s_f \left(\frac{1 - \frac{d}{t}}{1 - \frac{d}{tM}} \right) \quad (16)$$

$$\text{there is } P_p = 2s_f \frac{t}{D} \left(\frac{1 - \frac{d}{t}}{1 - \frac{d}{tM}} \right) \quad (17)$$

M is the Folias factor, and it is well known that the yield strength of the material, s_f , depends on the limit strength, s_y , by the relation:

$$s_f = m_t s_y \quad (18)$$

With s_y being the strength limit of the material and m_t a factor that for steel pipelines varies between 1.10 and 1.15. Further details of this variation are well described in Hopkins et al. (1992).

The application of the methods of structural reliability in pipelines depends on the definition of the limit state function. This function specifies the boundary between security and system engineering failure. In the case of pipelines, the limit state function can be specified by:

$$Z = P_p - P_a \quad (19)$$

where P_p , given by expression (17), corresponds to the pressure that the pipeline can withstand and P_a is the pressure that the fluid exerts on the walls of the duct.

3. Materials and Methods

3.1 Material: Real Data of two PIG runs

Real records of two PIG Palito runs in ducts of an oil company were used in this article. One of the runs refers to an extension corresponding to a safety position, that is, there was not a need for maintenance. The data corresponding to this situation form the set 1. The other run refers to an extension of the same length and which has already received maintenance whose data form the set 2.

Information obtained refers to occurrences of corrosion identified by loss of the pipe depth. It is important to record that these observations do not follow the distributions specified by Ahammed and Melchers (1996) in some of the variables, such as the outer diameter (D), multiplicative factor m_t and the yield strength from material s_y .

3.2 Methodology

In the problem studied there has been available $n = 14$ observations of the random vector $[d, D, L, M, P_a, s_y, t]$ for each one of the sets. Nevertheless, as some of the variables do not follow the probability distributions specified by Ahammed and Melchers (1996) and consequently there is no way to build them to run Monte Carlo method, we applied the Bootstrap technique. It should be noted that the use of this technique is independent from the probability distributions of the variables involved, because the Bootstrap estimates the true distribution from the data, without any priori assumption.

The parameters used to implement these techniques were as follows: 10,000 Bootstrap iterations (NBS = 10,000) and 100,000 Monte Carlo iterations (NMC = 100,000).

Operationally the process involved the following steps:

- 1) At first there is the original sample size $n = 14$, $[\underline{x}_1, \underline{x}_2, \dots, \underline{x}_{14}]$ with the records of corrosion, that is, 14 observations from the vector $[d, D, L, m_t, P_a, s_y, t]$. Variables that compose the vector are: depth of the defect d ; duct diameter D ; length of the defect L ; multiplication factor m_t ; fluid pressure applied to the duct P_a ; yield strength s_y and thickness of the duct wall t ; and the equation of limit state $Z = P_p - P_a$ with the pressure strength of the pipe given by:

$$P_p = 2m_t s_y \frac{t}{D} \cdot \frac{1 - \frac{1}{d}}{1 - \frac{1}{dM}} \tag{20}$$

with M being the Folias factor calculated by:

$$M = (1 + 0.6275 \frac{L^2}{Dt} - 0.003375 p_i^*) \quad p / \frac{L^2}{Dt} \leq 50 \tag{21}$$

$$M = 0.032 \frac{L^2}{Dt} + 3.3 \quad p / \frac{L^2}{Dt} > 50.$$

- 2) In the first iteration Monte Carlo, $i = 1$, it is taken the first Bootstrap sample from the original $[\underline{x}_1^*, \underline{x}_2^*, \dots, \underline{x}_{14}^*]$, then it is estimated the Bootstrap statistics P_{p1}^* and compares it to the P_{a1}^* value in the equation $Z = P_p - P_a$. It has to be registered the occurrence of failure or security with the values of the variable indicator $I = 1$ in the occurrence of failure or $I = 0$ in the security for each Bootstrap observation $k = 1, 2, \dots, 14$. Thus, it has to be estimated the probability of failure for the first sample Bootstrap, p_{f11}^* , from first Monte Carlo iteration considering:

$$I_k = \begin{cases} 1, & \text{failure occurred in the } k\text{-th observation} \\ 0, & \text{occurrence of success in the } k\text{-th observation} \end{cases}$$

$$p_{f11}^* = \frac{\text{(failure occurred in 1st Bootstrap Sample)}}{14} = \frac{\sum_{k=1}^{14} I_k}{14} \tag{22}$$

- 3) It is estimated the failure probability for each sample Bootstrap j , p_{f1j}^* $j = 1, 2, \dots$, NBS in the 1^a Monte Carlo iteration:

$$p_{f1j}^* = \frac{\sum_{k=1}^{14} I_k}{14} \tag{23}$$

- 4) Repeat steps 2 and 3 for $j = 1, 2, \dots$, NBS, always calculating the failure probability p_{fij}^* and obtaining the failure probability in Monte Carlo iteration i , $i = 1, 2, \dots$, NMC by:

$$p_{fi}^* = \frac{\sum_{j=1}^{NBS} P_{fij}^*}{NBS} \tag{24}$$

- 5) Finally, it is estimated the failure probability for all NMC Monte Carlo iterations, the standard error and a confidence interval for the estimated parameter respectively by the expressions (25), (26) e (27):

$$p_f^* = \frac{\sum_{i=1}^{NMC} P_{fi}^*}{NMC} \tag{25}$$

$$s^* = \sqrt{\frac{1}{NMC-1} \sum_{i=1}^{NMC} [p_{fi}^* - P_f^*]^2} \tag{26}$$

$$[P_{f(B+1)(\alpha/2)}^* ; P_{f(B+1)(1-\alpha/2)}^*] \tag{27}$$

Bootstrap failure probability values p_{fi}^* , form the Bootstrap distribution of that statistic. Results obtained by the software are shown in Table 1 in the results section.

4. Results

4.1 Results for Real Data Set 1

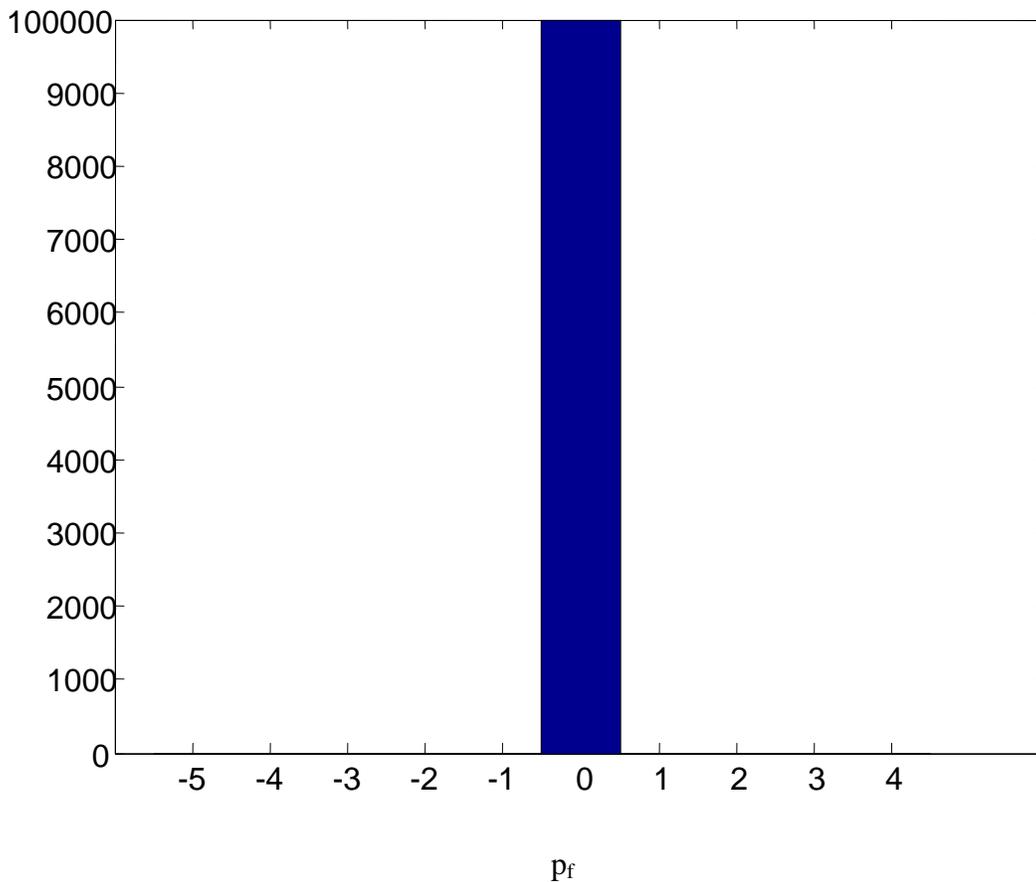
Data entry was done using EXCEL file and on the initial screen it was considered 14 observations (sample n = 14), 10,000 iterations Bootstrap (NBS = 10,000), 100,000 Monte Carlo iterations (NMC = 100,000). Results provided by the software are shown in Table 1.

Table 1 – Results of Analysis by Monte Carlo and Bootstrap - Set 1

P_f^*	s^*	$P_{0,025}^*$	$P_{0,975}^*$
0.0000000000000000	0	0	0

It has been observed that the numbers show the situation of complete security. There has been no failure in 100,000 Monte Carlo replications. This fact expresses no need for close maintenance. The correspondent histogram to Bootstrap values of pf obtained is shown in Figure 5.

Figure 5 - Histogram of Values Obtained For Bootstrap p_f -Set 1



4.2 Results for Real Data Set 2

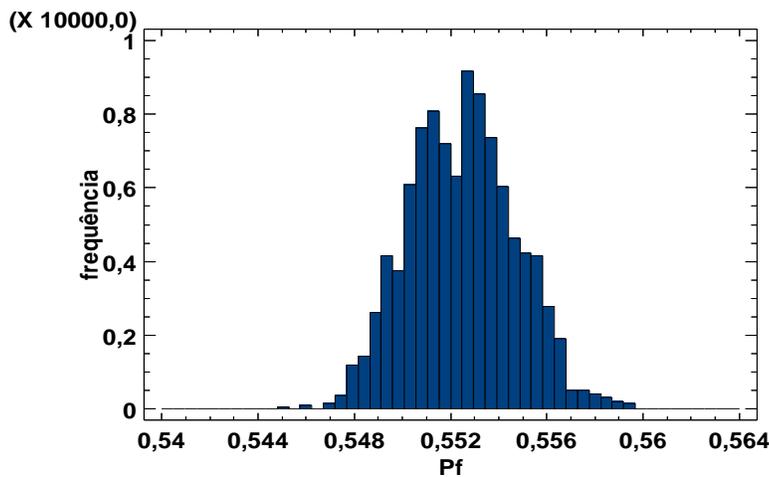
Data entry was performed using EXCEL file on the initial screen and there were considered 14 observations (sample $n = 14$), 10,000 iterations Bootstrap (NBS = 10,000), 100,000 Monte Carlo iterations (NMC = 100,000). Results obtained by the software are shown in Table 2.

Table 2 - Analysis Results by Monte Carlo and Bootstrap - Set 2

p_f^*	s^*	$P_{0,025}^*$	$P_{0,975}^*$
0,55250	0,00221508	0,54816	0,55684

Analyzing the results in Table 2, it is possible to observe an estimated failure probability of 55.25% and the confidence interval of 95% level for the true failure probability is [54.816%; 55.684%], that is, $P(54,816\% < p_f < 55,684\%) = 95\%$. In practical terms, the results found are relevant for a better planning of maintenance of such pipelines. Histogram shown in Figure 6 corresponds to the Bootstrap values of p_f obtained.

Figure 6 - Histogram of Bootstrap Values Obtained for p_f Set 2



5. Conclusion

Application of structural reliability in pipelines and the use of Monte Carlo method associated with the Bootstrap, by the simplicity and the fact that the calculation does not depend on assumptions about the nature of the probability distributions of the variables involved, it has shown reasonable results. It was obtained the parameter evaluation by point and interval as well, since there has been measured the statistical variability. Monte Carlo method, associated with Bootstrap was applied in two runs of the PIG; one was known as a safe situation and therefore with diminished probability of failure. It was known that the other had gotten maintenance, however there was not a measure of its probability of failure. Results of applying the methods were consistent with this information. It is important to emphasize that the real data of the two runs available did not verify the distribution of specified probability for some of the variables. In this situation, the application of Bootstrap enabled the use of Monte Carlo method.

References

- Ahamed, M., Melchers, R. E., 1996. Reliability estimation of pressurized pipelines subject to localized corrosion defects. *Int. J. Pres. Ves. Piping*, n. 69, p. 267-272.
- Ahamed, M., 1997. Prediction of Remaining Strength of Corroded Pressurized Pipelines. *Int. J. Pres. Ves. Piping*, n. 71, p. 213-217.
- Ahamed, M., 1998. Probabilistic Estimation of Remaining Life of a Pipeline in the Presence of Active Corrosion Defects. *Int. J. Pres. Ves. Piping*, n. 75, p. 321-329.
- Cabral, H. L. D., 2007. Desenvolvimento de Ferramentas Computacionais para Modelagem e Análise Automática de Defeitos de Corrosão em Dutos. Dissertação de Mestrado, Universidade Federal de Pernambuco, Recife.
- Choi, J. B., Goo, B. K., Kim, J.C., 2003. Development of Limit Load Solutions for Corroded Gas Pipelines, *Int. J. Pres. Ves. Piping*, v.80, 2003, p.121-128.
- Cordeiro, A. L. R., 2008. Aplicação dos Métodos FORM e SORM para Avaliação de Dutos Corroídos. Dissertação (Mestrado em Engenharia Mecânica), Pontifícia Universidade Católica do Paraná, Curitiba.
- Cordeiro, A. L. R., Machado, R. D., Abdalla Filho, J. E., 2009. Aplicação dos Métodos FORM e SORM para Avaliação de Dutos. In: CILAMCE - Iberian-Latin-American Congress on Computational Methods in Engineering, 30th, Búzios. Proceedings of 30th CILAMCE- Iberian Latin American Congress on Computational Engineering, Rio de Janeiro.
- Ditlevsen, O., Madsen, H.O., 1996. *Structural Reliability Methods*. J. Willey & Sons, New York, 1996.
- Efron, B., 1979. Bootstrap methods: another look at Jackknife. *Annals of Statistics*, v. 7, n. 1, p. 1-26.
- Freudenthal, A., 1947. The Safety of Structures. *Transactions of the American Society of Civil Engineers*, vol. 112, n. 1, January 1947, pp. 125-159.
- Hopkins, P., Jones, D. G., 1992. A Study of the behavior of long and complex-shaped corrosion in transmission pipelines. In: *Proc. Int. Conf. On Offshore Mechanics And Arctic Engineering*, 11th, ASME v. V, Part A, pp. 211-217.
- Kijewski, T., Kareem, A., 2002. On the reliability of a class of system identification techniques: insights from Bootstrap theory, *Structural Safety*, 24, p. 261-280.
- Melchers, R. E., 1987. *Structural Reliability: analysis and prediction*. Ellis Horwood Limited, Chichester.
- Pedroni, N., Zio, E., Apostoloski, G.E., 2010. Comparison of bootstrapped artificial neural networks and quadratic response surfaces for the estimation of the functional failure probability of a thermal-hydraulic passive system, *Reliability Engineering and System Safety*, 95, p. 386-395.
- Picheny, V. Nam, H. K., Haftka, R. T., 2010. Application of Bootstrap method in conservative estimation of reliability with limited samples, *Struct Multidisc Optm*, 41, p. 205-217.
- Squarcio, R. M. F., Chaves Neto, A., 2009. Análise da Confiabilidade de Oleodutos Corroídos Utilizando o Método de Monte Carlo. *Anais do CNMAC*, Cuiabá, v. 2, p. 621-627.
- Vanhazebrouck, V. M., 2008. Análise de Dutos Corroídos por Meio de Método de Confiabilidade Estrutural. Dissertação de Mestrado, Pontifícia Universidade Católica do Paraná, Curitiba.