The Connection between the Binary Search Tree and the Dynamical Systems Trees

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Abstract

In this paper we will study the connection between binary search tree and the dynamical system trees.

Keywords: Hidden Markov model on binary search tree , Markov chain on binary search tree

Introduction

The dynamical systems trees was introduced by Andrew Howard and Tony Jebara that consider dynamical systems trees (DSTs) as a flexible class of models for describing multiple processes that interact via a hierachy of aggregating parent chains [1].A hidden Markov model is a tool for representing probability distribution over sequences of observations [2].we will introduce a way for representing the interacting processes.

Algebra

Definition1

In computer science, a binary search tree is a binary tree that has the properties that the left sub tree of a node contains only nodes with keys less than the key of node. The right sub tree contains only nodes with keys greater than the key of node. The Right and left sub trees must be also binary search trees.There must be no duplicate nodes.

Definition 2

The dynamical systems trees is a model for describing multiple processes that interact via a hierarchy of aggregating parent chains , a leaf processes is modeled as a dynamical system containing discrete and/ or continuous hidden state with discrete and/or Gaussian emissions which is called switching linear dynamical system[3].

Main Result

We first show how to define Markov chains on binary search trees, consider a binary search tree as shown in figure 1, in Markov chain each state depends on the previous state (or any state in the future depends on the current state)

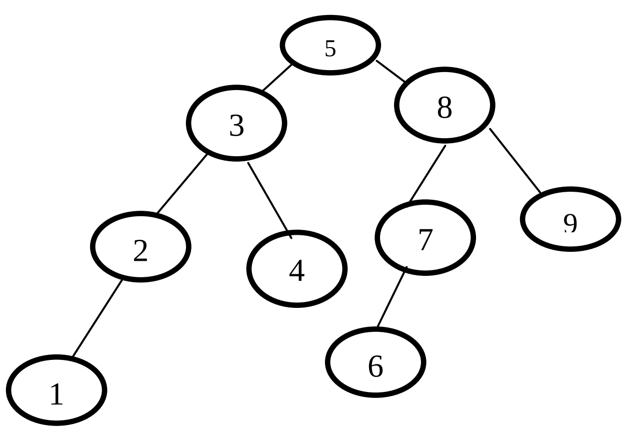


Fig. 1: We Convert this Tree into a Directed Tree as Follows in Figure 2

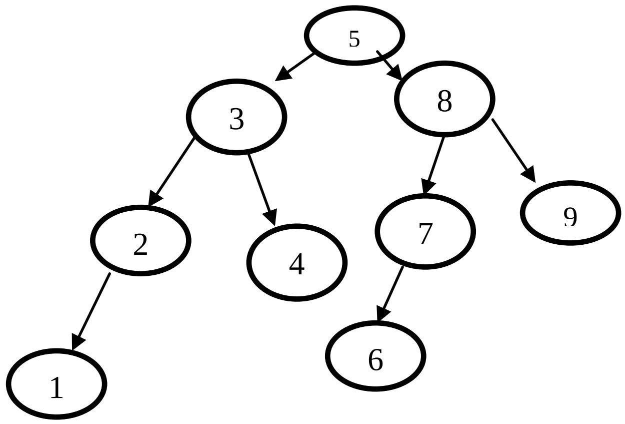


Fig.2

Since the tree is connected and each node in any level of this tree receive an edge from nodes in previous level and we can estimate that the nodes in any level from that fact that the vertices to right have keys greater to vertices to left .Thus we deal with a Markov chain in which every level depends on the previous level, thus we consider that the states of Markov chain as the levels of the tree.

How we can define the Interacting between Processes by Binary Search Trees?

To answer this question, factorial hidden Markov models [3] and coupled hidden Markov models describe the interact between multiple hidden Markov models.

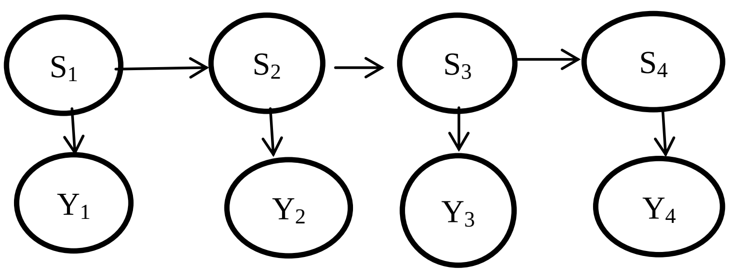


Fig.3: In Figure 3 we represent the Directed Acyclic Graph for HMM

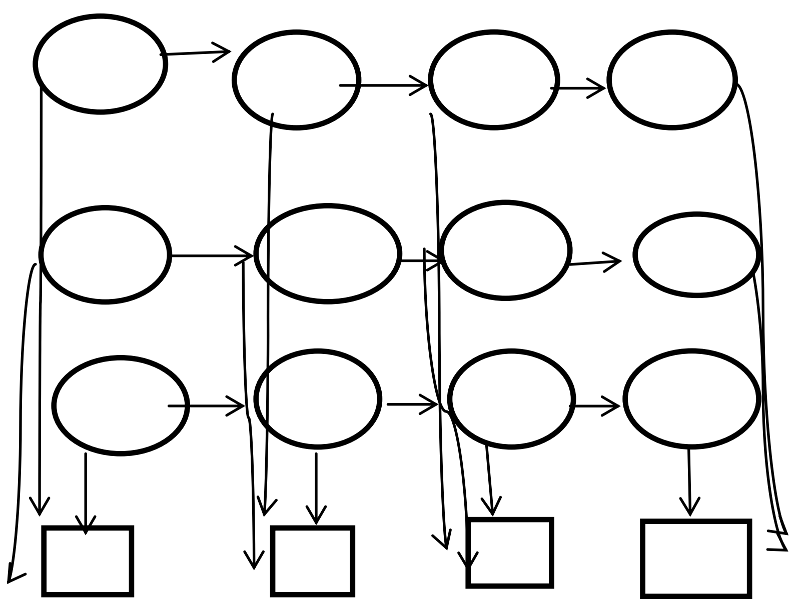


Fig 4

Figure 4 is a graph representing the factorial hidden Markov models .

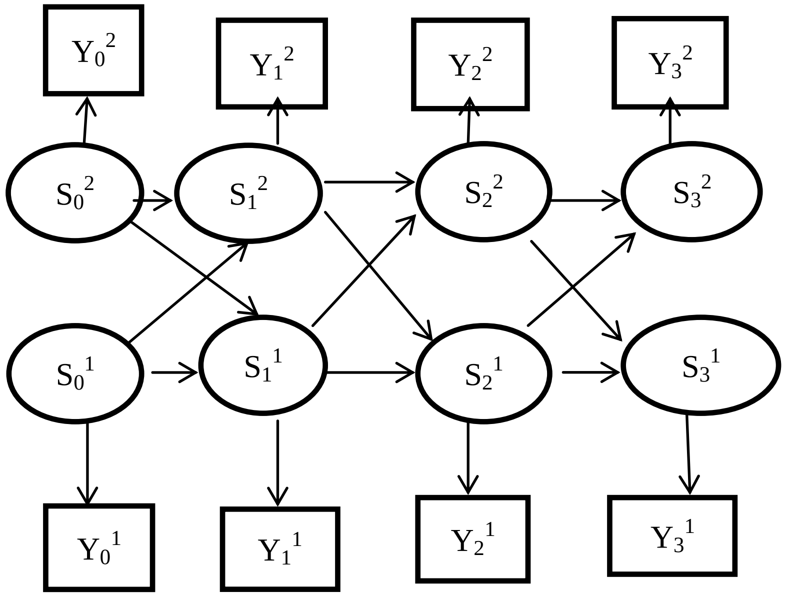


Fig.5

Figure 5 a graph for coupled hidden Markov models that represent the interaction between multiple hidden Markov models through different emission streams.

If we assume that the levels of binary search tree are hidden and we can estimate the number of levels from the information that in the first level it contain only node which is
root and the second contain two for example .

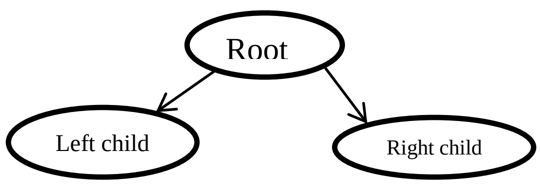


Fig .6 .A Binary search tree

This discussion can put in the following figure

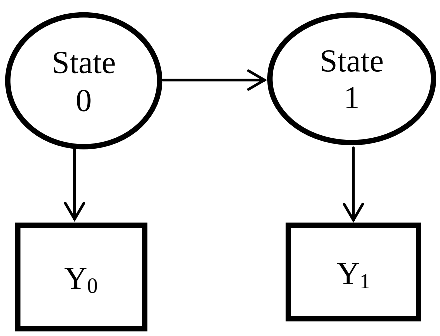
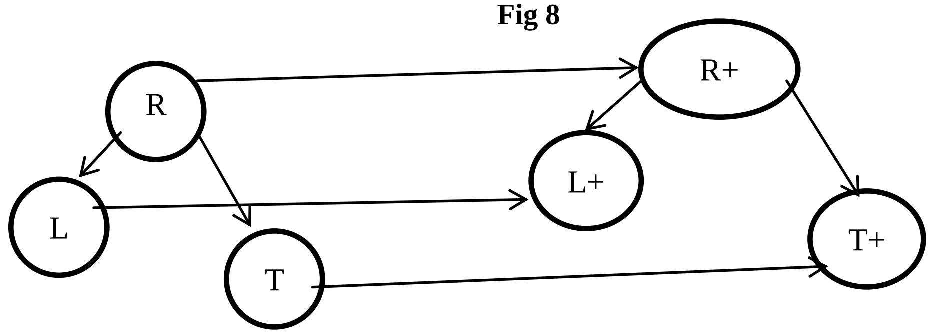


Fig 7: Directed Acyclic Graph represent the relation between States of Binary Search Tree and the number of vertices of each level (State) and by this way we can define a HMM on Binary Search Tree

Two binary search tree of this property can interact if the root of one is greater than the root of the other , and the same thing with the left and right sub trees .Here in the following figures we deal with a binary search trees with only two levels .

Fig 8



In figure 8 if the condition satisfies two binary search trees of this form can occur , and can be expressed in the following network:

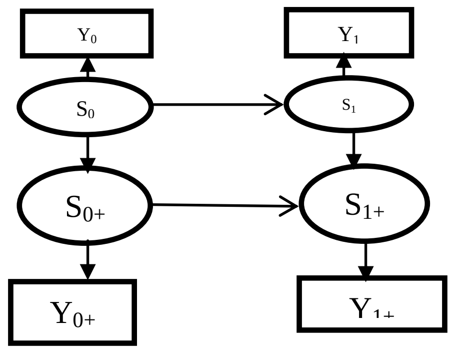


Fig 9: A Directed Acyclic Graph representing the interaction between two Binary Search Trees

And the thing with 3 binary search trees defined on HMM:

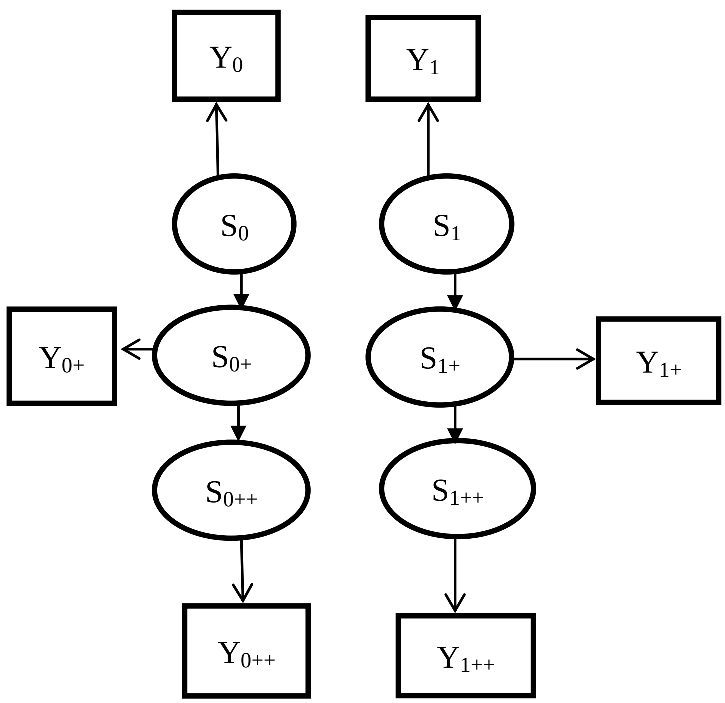


Fig 10: A Directed Acyclic Graph represents the interact between 3 Binary Trees at the first and second level of each tree.

As we knew before that the dynamical systems trees is a flexible class for describing the interacting between multiple processes and the leaf process is a switching linear dynamical systems ,in our case we consider the dynamical systems trees as a hierarchy of aggregating parent chains and the leaf process at the lowest level is consider as a binary search tree defined on hidden Markov models with condition of interaction described above. This can show graphically as follows:

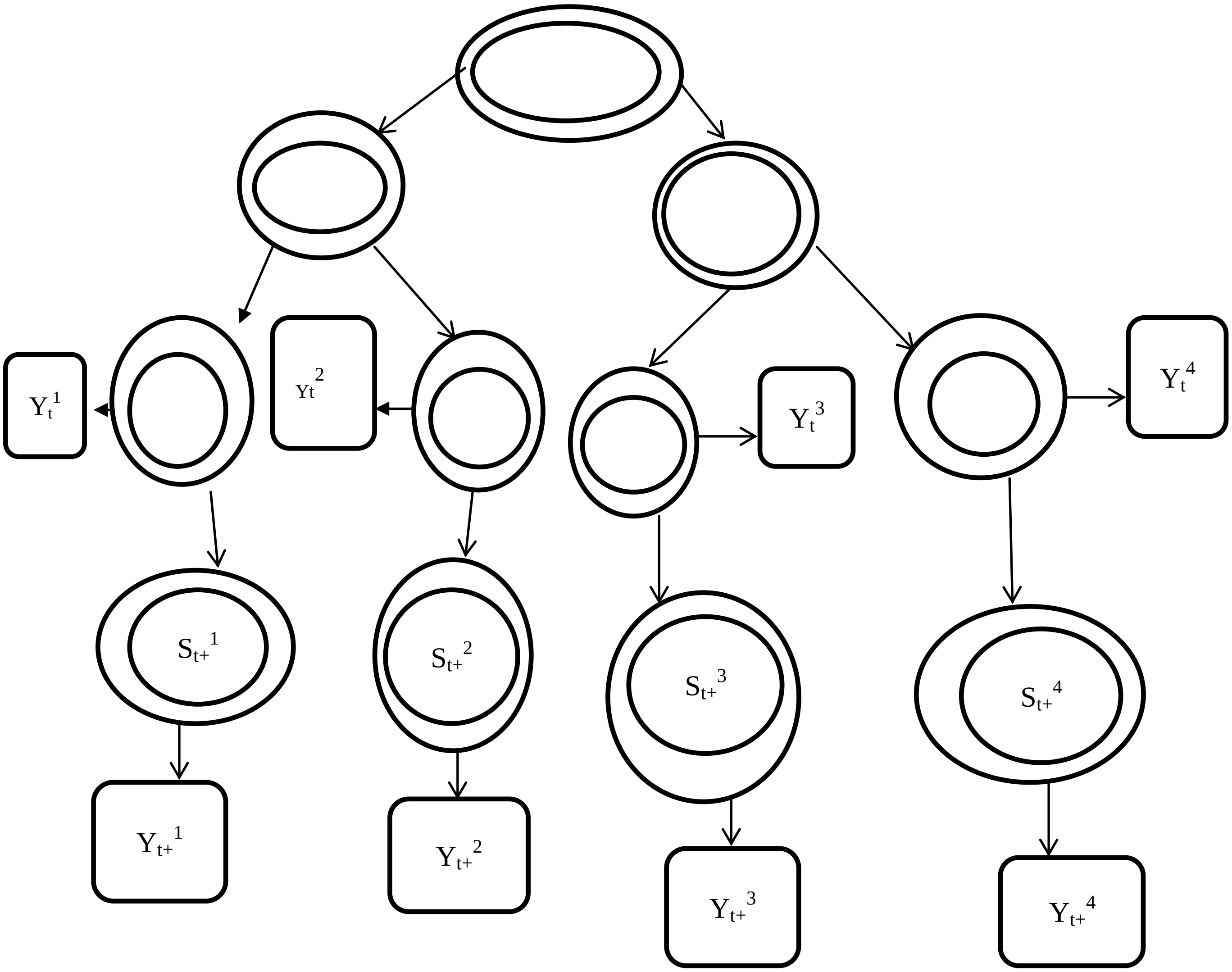


Fig 11: Dynamical Systems Trees (With Replicators) with Lowest Level (Leaf Process a Binary Search Trees Defined on Hidden Markov Models

References

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