A Novel Numerical Approach to the Dynamics Analysis of Marine Cables

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Abstract

Marine cables are exposed to the action of water in relative motion, and hence the need to predict the dynamic behavior of these complex mechanical systems under the typical working conditions since the outset of the corresponding design processes. This paper describes the basic principles that can be used in a computer program meant to support the design of marine cables that should work under a combination of static and dynamic loadings. A unique mathematical model and a matching numerical method based on finite differences are used, first for determining the static equilibrium configuration of the riser, and then for findings its dynamic response around the formerly computed static configuration. A computer program was specially developed for implementing the proposed model, and a step-by-step method for the design of marine cables similar to the ones in operation at the Black Sea are herein presented. The conclusion is that this new model provides a coherent and efficient means to analyze the dynamic behavior of marine cables, as required during the design process.

Key Words: marine cables; dynamic behavior; finite differences; optimization

1. Introduction

The main objective of this study is to deploy a coherent and efficient mathematical model for the cable mechanics. Beginning from this of this model will be determinate the cable/chains mechanics for different technical situations. Starting from this model, the cable behavior for different technical situations is analyzed, and the corresponding original computer programs were used to determine the cable's mechanics. The marine cable/chains utilization presents actually a great diversity including:

1. Moored systems: vessels, offshore exploration units, surface or underwater body, floating dock, other floating structures; 2. Towed systems: surface vessels, underwater body, fishing nets.

Problem Formulation

A flexible towing/mooring cable presents a difficult problem in dynamic analysis so looking the mathematical model formulation but especially concerning the numerical computation.

By making key assumptions is possible to obtain a satisfactory accuracy of mathematical model and of computation results respectively. Several mathematical models are used to describe the dynamical behavior of cable. In the present time our experience is confirmed relative to two principal methods, which are applied in the design of moored/towed systems. The first approach taken for moored/towed systems prediction has been to represent the continuous cable with a series of rigid segments having the mass-elastic characteristic concentrated at discrete points along the cable. It is a development of the lumped-mass method [7]. The equations of motion are obtained by expressing the condition of dynamical equilibrium – D'Alembert principle- for each cable element or these equations of motion are derived by Lagrange's equations. One can mention the analyses of Walton & Polachek were based on the use of a lumped characteristics model [12]. In 1980, Nakajima *et al.* developed a new method based on a lumped mass model that can be used for the complete analysis — both static and dynamic — of cable mechanics [10].

The second approach taken for moored/towed systems prediction has been to represent the marine cable as a continuous medium having the mass-elastic characteristic distributed along the cable. The cable motion is described by a system of partial differential equations. Wicker, L.E. obtained a solution for these equations using the method of characteristic based on the partial differential equations of the cable motion in a bi-dimensional space [1]. Bernitsas, M. M. developed a study for a three dimensional nonlinear large deflection model for dynamic behavior of risers, pipelines and cables [4]. In addition, several researchers have developed mathematical models to take in account the bending stiffness and torsion stiffness [5, 6], as well as more accurate expressions for the hydrodynamic forces and for the mass distribution [6]. Some other methods for the mooring/towing cables design were developed [2, 3, 5, 6, 7, 10, 11]. Several studies on cable systems have been made by our R&D group. The present paper reports the basic approach that has been made on modeling a mooring/towing system, on deriving the equations for the marine cable mechanics, and on the introducing the necessary engineering assumptions to achieve practical solutions.

Moored/Towed Systems

The approach taken for moored/towed system has been to represents the continuous cable composed of elastic segments "ds" having mass and elastic characteristic distributed along the elements. As such the mechanical model of cable dynamics approximates the set of exact partial differential equations with another set of non-linear ordinary differential equations because they are amenable to direct solution by computation. The assumptions can be summarized as follow:

1. The cable is represented by "n" rigid segments;
2. The mass-elastic characteristic of cable is distributed along the each segment;
3. The elongation of the cable is considered;
4. Accelerations arising from changes in the towing ship's turning rate or speed are assumed to be negligible;
5. Accelerations due to the products of angular rates-gyroscopic effect- are assumed to be negligible;
6. Dynamic coupling between model segment due to inertial effects can be neglected;
7. The towline length can be constant or variable respectively.

The hydrodynamic forces acting on each element are determinates by using the relative velocity distribution along each element. These forces are reduced at the discrete points of cable. The hydrodynamic forces are nonlinear functions of the relative water velocities of each segment.

2. The Dynamics of Marine Cable in a Bi-Dimensional Space

In several cases, both the ends of the cable and the water flow are located in the same vertical plane (Figure 1). Also the cable excitation is made in the same plane. In those cases, the cable motion is performed in the same vertical plane. As a general feature of our approach we have developed a mathematical model for dynamic analysis of cable motion from which, by particularizing, one can deducts the model of marine cable equilibrium configuration (Figure 1). The marine cable equilibrium configuration is needed for the complete analysis of marine cable motion, because it determines the initial configuration of the cable, which is considered an equilibrium configuration. A broad model has been developed to support the prediction of the cable motion in an offshore operation condition. The model is composed by the differential equations of the cable motion, and the related methods can be used in several specific situations:

1. Single or compound moored systems (Composed by cables, chains, underwater or surface bodies, buoys, weights and docks.);
2. Distinct boundary conditions at both the lower and the upper ends;
3. Different loads, both concentrated and/or distributed;
4. Different continuity condition in the points where bodies are located, or in the points where distinct cables are coupled;
5. Distinct bi-dimensional and three-dimensional geometric configurations;
6. Diverse water depth;
7. Diverse seabed topography;
8. Single or compound mooring/towing systems.

The solutions can be obtained in two different cases:

1. The boundary conditions are imposed at both ends of the cable. Only a part of the configuration parameters is known, and the remaining parameters are determined through computation;
2. The boundary conditions are imposed only at one of the cable ends.

All the parameters must be known in the cable end where the computation begins. The conditions that are required for stopping the computation are imposed in the other cable end.

Kinematics Relationships

Taking into account the fact that the differential equations of cable motion will be solved by integrating step by step on time, the relationship between speed, velocity and displecemts must be expressed as it follows.
If the index of cable point M is noted by „υ" and the index of cable point M_1 is noted by „υ+1" (Figure 2a) then we can writte.

For the current point of cable, M, we can note:

a. Position vector of point M in the coordinates system xOz:

$$(r_{\nu,t}) = (x_{\nu,t} \quad z_{\nu,t})^T; \ (r_{\nu,o}) = (x_{\nu,o} \quad z_{\nu,o})^T \tag{1}$$

where $(r_{\nu,t})$ defines the current position of point M at the instant t in the coordinate system xOz, and $(r_{\nu,o})$ defines the initial position of point M relative to the coordinate system xOz; $x_{\nu,t}, z_{\nu,t}$ are the coordinates of the current point, M, in the coordinate system xOz at the current time, t; and $x_{\nu,o}, z_{\nu,o}$ are the coordinates of the current point, M, in the coordinate system xOz, at the initial time, t_o, as found during the previous static analysis.

b. Velocity vector of point M at time t+Δt in the coordinate system xOz:

$$(\dot{r}_{\nu,t+\Delta t}) = (\dot{x}_{\nu,t+\Delta t} \quad \dot{z}_{\nu,t+\Delta t})^T \tag{2}$$

and velocity vector at time t:

$$(\dot{r}_{\nu,t}) = (\dot{x}_{\nu,t} \quad \dot{z}_{\nu,t})^T \tag{3}$$

where Δt is the time step for the displacement of point M.

The components of the velocity vector at time t+Δt, can be expressed as:

$$\dot{x}_{\nu,t+\Delta t} = \dot{x}_{\nu,t} + \frac{x_{\nu,t+\Delta t} - x_{\nu,t}}{\Delta t}; \ \dot{z}_{\nu,t+\Delta t} = \dot{z}_{\nu,t} + \frac{z_{\nu,t+\Delta t} - z_{\nu,t}}{\Delta t} \tag{4}$$

where $x_{\nu,t+\Delta t}$ and $z_{\nu,t+\Delta t}$ are the coordinates of current point M, at time t+Δt.

c. The acceleration vector of point M in the coordinate system xOz, at time t+Δt , can be expressed as

$$(\ddot{r}_{\nu,t+\Delta t}) = (\ddot{x}_{\nu,t+\Delta t} \quad \ddot{z}_{\nu,t+\Delta t})^T \tag{5a}$$

The same vector at time t can be expressed as:

$$(\ddot{r}_{\nu,t}) = (\ddot{x}_{\nu,t} \quad \ddot{z}_{\nu,t})^T \tag{5b}$$

The components of the vector (5a) can be expressed as:

$$\ddot{x}_{\nu,t+\Delta t} = \ddot{x}_{\nu,t} + \frac{\dot{x}_{\nu,t+\Delta t} - \dot{x}_{\nu,t}}{\Delta t};\ \ddot{z}_{\nu,t+\Delta t} = \ddot{z}_{\nu,t} + \frac{\dot{z}_{\nu,t+\Delta t} - \dot{z}_{\nu,t}}{\Delta t} \tag{5c}$$

The projections of the velocity and of the acceleration of the midle point of the element „ds" (Figure 2), can be written as

$$x_i = \frac{x_{\nu+1} + x_\nu}{2};\quad z_i = \frac{z_{\nu+1} + z_\nu}{2} \tag{6a}$$

$$v_x(z_i) = \dot{x}_i = \frac{\dot{x}_{\upsilon+1} + \dot{x}_\upsilon}{2};\ v_z(z_i) = \dot{z}_i = \frac{\dot{z}_{\upsilon+1} + \dot{z}_\upsilon}{2} \tag{6b}$$

$$\ddot{x}_i = \frac{\ddot{x}_{\upsilon+1} + \ddot{x}_\upsilon}{2};\ \ddot{z}_i = \frac{\ddot{z}_{\upsilon+1} + \ddot{z}_\upsilon}{2} \tag{6c}$$

$$(a_c)_t = (\ddot{x}_{i,t}\quad \ddot{z}_{i,t})^T \tag{6d}$$

i=1,...,n; ν =1,...,n; where n is the number of riser elements.

The velocity of the riser relative to the water in the coordinate system xOz (see Figure 2b) can be written as:

$$\mathbf{v_r}(z_i) = [[v_c(z_i)\sin\gamma] + v_x(z_i)]\mathbf{i} + [[-v_c(z_i)\cos\gamma] + v_z(z_i)]\mathbf{k} \tag{7a}$$

where $v_c(z_i)$ is the water current velocity in the midle point of the current element „ds", (Figure 2), and $v_x(z_i)$, $v_z(z_i)$ are the components of the velocity vector $(v(z_i))$ at the midle point of element „ds" relative to the inertial coordinates system xOz.

One should notice that:

$$(v(z_i)) = (v_x(z_i)\quad v_z(z_i))^T \quad ; \quad (v_c(z_i)) = (v_{cx}(z_i)\quad v_{cz}(z_i))^T \tag{7b}$$

Because the parameters of the next riser point are unknown during the iterative computation of the equilibrium riser configuration, the expression of the relative velocity is written as:

$$\mathbf{v_r}(z_\nu) = [v_c(z_\nu)\sin\gamma]\mathbf{i} + [-v_c(z_\nu)\cos\gamma]\mathbf{k} \tag{8}$$

The relative velocity unit vector, corresponding to the middle point of element „ds" can be expressed as:

$$\mathbf{u}(z_i) = \frac{\mathbf{v_r}(z_i)}{|\mathbf{v_r}(z_i)|} = u_x(z_i)\mathbf{i} + u_z(z_i)\mathbf{k} \tag{9a}$$

and the relative velocity unit vector in the current riser point is:

$$\mathbf{u}(z_\nu) = \frac{\mathbf{v_r}(z_\nu)}{|\mathbf{v_r}(z_\nu)|} = u_x(z_\nu)\mathbf{i} + u_z(z_\nu)\mathbf{k} \tag{9b}$$

Where

$$\begin{aligned} u_x(z_i) &= [v_c(z_i)\sin\gamma(z_i) + v_x(z_i)]/|\mathbf{v_r}(z_i)| \\ u_z(z_i) &= [-v_c(z_i)\cos\gamma(z_i) + v_z(z_i)]/|\mathbf{v_r}(z_i)| \end{aligned} \tag{10a}$$

For determining the equilibrium configuration, we will use:

$$\begin{aligned} u_x(z_\nu) &= [v_c(z_\nu)\sin\gamma(z_\nu)]/|\mathbf{v_r}(z_\nu)| \\ u_z(z_\nu) &= [-v_c(z_\nu)\cos\gamma(z_\nu)]/|\mathbf{v_r}(z_\nu)| \end{aligned} \tag{10b}$$

The hydrodynamic forces acting on the riser unit length (Figure 2a) are expressed in the coordinate system of unit vectors □, **n**, which expressions are:

– Tangent unit vector, □: $\boldsymbol{\tau} = \tau_x\mathbf{i} + \tau_z\mathbf{k} = \cos\theta\mathbf{i} + \sin\theta\mathbf{k}$

– Normal unit vector, **n**: $\mathbf{n} = n_x\mathbf{i} + n_z\mathbf{k} = \sin\theta\mathbf{i} - \cos\theta\mathbf{k}$

In the system of unit vectors τ, **n**, we have the following expressions for the hydrodynamic forces acting on the cable unit length [1]:

$$F_\tau = \frac{1}{2}\rho_w\, d_e\, c_\tau v_r^2 \,[0.083\cos(\mathbf{v_r},\boldsymbol{\tau}) - 0.035\cos^2(\mathbf{v_r},\boldsymbol{\tau})]$$

$$F_n = \frac{1}{2}\rho_w\, d_e\, c_n v_r^2 \left|\sin(\mathbf{v_r},\boldsymbol{\tau})\right| \sin(\mathbf{v_r},\boldsymbol{\tau}) \tag{11}$$

where cos($\mathbf{v_r}$, τ) = $\mathbf{u}$τ, ρ_w is the water density, c_τ, c_n are the hydrodynamic coefficients for the bare cable [1], and d_e is the external diameter of the cable hosepipe.

One can neglect the effect of the frictional forces when the water flow velocity is low. According the Pode's testing data [1], usually one can neglect the tangential component of the hydrodynamic force. The hydrodynamic forces for the cable element can be expressed in the inertial coordinates system xOz using the matrix notation. Denoting by (F) the vector of the hydrodynamic forces in the system of unit vectors τ, **n**, and by (f) the vector of the same forces in the coordinate system xOz, we can notice that

$$(F) = (F_\tau \quad F_n)^T \;;\; (f) = (f_x \quad f_z)^T \tag{12}$$

And

$$(f) = [R](F) \tag{13}$$

where [R] is the transformation matrix between the system of unit vectors τ, **n**, and the coordinate system xOz:

$$[R] = \begin{bmatrix} \tau_x & n_x \\ \tau_z & n_z \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \tag{14}$$

For the tension at both ends of the riser element "ds", we have:

$$(T) = [R](T_L) \tag{15}$$

where:

$$(T) = (T_x \quad T_z)^T \tag{16}$$

$$(T_L) = (-T \quad 0)^T \tag{17}$$

At the lower end of element "ds", there is

$$(T_1) = [[R] + d[R]](T_{1L}) \tag{18}$$

where d[R] is the total differential of the matrix [R], the second and higher-order infinitesimals being neglected. In addition:

$$(T_{1L}) = (T + dT \quad 0)^T \tag{19}$$

To apply the D'Alembert's principle we need to know the vector of inertial forces. The mapping of the vectors of inertial force between the coordinate system xOz as (F_i) and the system of versors τ, **n**, *i.e.* (F_{iL}), is given by:

$$(F_i) = (F_{in}) + [R](F_\lambda) \tag{20}$$

Where

$$(F_i) = (F_{ix} \quad F_{iz})^T \;;\; (F_{in}) = (F_{in,x} \quad F_{in,z})^T \;;\; F_{in,x} = \rho_c \ddot{x}_i \;;\; F_{in,z} = \rho_c \ddot{z}_i \tag{21}$$

with ρ_c being the mass per unit length of the cable.

The vector of added mass per unit lenght of the cable is

$$(F_\lambda) = (F_{\lambda_\tau} \quad F_{\lambda_n})^T, \tag{22}$$

which elements in the the system of unit vectors τ, **n** are given by

$$F_{\lambda_\tau} = \lambda_\tau a_\tau \;;\; F_{\lambda_n} = \lambda_n a_n \tag{23}$$

where the acceleration projections in the the system of unit vectors □, **n**, are a□ and a_n, and

$$\lambda_n = \lambda_p = \frac{1}{2}\rho_w\, c_{in}\, \pi d_e^2; \quad \lambda_\tau = \frac{1}{2}\rho_w\, c_{i\tau}\, \pi d_e^2, \tag{24}$$

ρ_w being the mass density of water, d_e the external diameter of the cable, and $c_{in}, c_{i\tau}$ the inertial coefficients that are determined through experimental tests.

Based on the expression

$$(a)_t = [R]^T (a_c)_t \tag{25}$$

where the $(a)_t$ can be expressed by the relationship:

$$(a)_t = (a_\tau \quad a_n)_t^T \tag{26}$$

one can write:

$$\begin{aligned} a_\tau &= \tau_x \ddot{x}_{i,t} + \tau_z \ddot{z}_{i,t} = \cos\theta(t)\ddot{x}_{i,t} + \sin\theta(t)\ddot{z}_{i,t} \\ a_n &= n_x \ddot{x}_{i,t} + n_z \ddot{z}_{i,t} = \sin\theta(t)\ddot{x}_{i,t} - \cos\theta(t)\ddot{z}_{i,t} \end{aligned} \tag{27}$$

Now, applying the D'Alembert's principle, the dynamic equilibrium equation of element "ds" can be written as:

$$(T) + (T_1) + (f)ds + (P)ds + (F_i)ds = 0 \tag{28}$$

where "ds" is the initial length of the cable element. The weight vector, (P), for the unit length of the cable is:

$$(P) = (0 \quad q)^T \tag{29}$$

where q is the weight in water per unit cable length, and (T), (T_1) are the cable tension vectors at both ends of the cable element. Thus, the differential equations of cable motion can be written as:

$$\begin{aligned} &\frac{dT}{ds} = (a_1 - e_1)c_T; \quad \frac{d\theta}{ds} = (\frac{a_2}{T} + \frac{e_2}{T})c_T; \\ &\frac{dS}{ds} = c_T; \quad \frac{dx}{ds} = c_T \cos\theta; \quad \frac{dz}{ds} = c_T \sin\theta; \quad c_T = 1 + \frac{T}{EA} \end{aligned} \tag{30}$$

where:

$$a_1 = q\sin\theta + \rho_c(\ddot{x}_i \cos\theta + \ddot{z}_i \sin\theta); \; a_2 = q\cos\theta - \rho_c(\ddot{x}_i \sin\theta - \ddot{z}_i \cos\theta)$$

$$e_1 = F_\tau + \lambda_\tau a_\tau; \quad e_2 = F_n + \lambda_n a_n$$

After the integration of the Eq. (30) one can integrate the equations

$$\dot{x} = \frac{dx}{dt}; \dot{z} = \frac{dz}{dt}; \ddot{x} = \frac{d\dot{x}}{dt}; \ddot{z} = \frac{d\dot{z}}{dt} \tag{31}$$

The Static Equilibrium Configuration

In the specific case of the equilibrium configuration, by particularizing Eq. (30)

$$a_1 = q\sin\theta; \; a_2 = q\cos\theta; \; e_1 = F_\tau; \; e_2 = F_n$$

one can write the following differential equations:

$$\begin{aligned} &\frac{dT}{ds} = (q\sin\theta - F_\tau c_T); \; \frac{d\theta}{ds} = (\frac{q}{T}\cos\theta + \frac{F_n}{T})c_T; \\ &\frac{dx}{ds} = c_T \cos\theta; \; \frac{dz}{ds} = c_T \sin\theta; \; \frac{dS}{ds} = c_T \end{aligned} \tag{32}$$

The non-linear differential Eqs. (32) are defined in the global coordinate system xOz (Figure 1), where s is cable arc (for the inextensible cable) at the current point, as measured from point O; S is cable arc (for the elastic cable) at the current point, as measured from the same point O; q is the weight (in water) of the unit cable length; x, z are the current point coordinates;

T is the cable tension at the current point; θ is the angle between the x-axis and the tangent unit vector at the current point; and F□ □ F_p are the tangential and normal components of the hydrodynamic force per unit length of the cable (Figure 2a). The expressions for both these components are the following:

$$F_\tau = \frac{1}{2}\rho_w\, d_e\, c_\tau v(z_v)\cos^2\theta\left|v(z_v)\right| \;;\; F_n = \frac{1}{2}\rho_w\, d_e\, c_n v(z_v)\sin\theta\left|v(z_v)\sin\theta\right| \qquad (33)$$

where ρ_w is the water density; d_e is the characteristic diameter of the cable; c□ □ c_n are the hydrodynamic coefficients (c□ ≈ 0□ c_n = 1); and $v(z_v)$ is the flow velocity corresponding to the coordinate z_v of the cable's current point. Due to the infinitesimal length of the line element, "ds", it is assumed that the water flow velocity is uniform along the element, and it is equal to the flow velocity that corresponds to the z-coordinate of the cable's current point, M (see Figure 2).

The solutions for Eq. (30) and Eq. (33) are obtained by numerical integration after they are transformed in an algebraic system – Eq. (34) and Eq. (35) - by using the finite-difference method with "h" step size.

2.1- For the Cable Motion in the Plane Xoz:

$$\begin{aligned}
&T_j(t+\Delta t) = T_{j-1}(t+\Delta t) + h(a_{1,j-1} - he_{1,j-1})c_T \\
&\theta_j(t+\Delta t) = \theta_{j-1}(t+\Delta t) + \frac{h}{T_{j-1}}[a_{2,j-1} + e_{2,j-1}]c_T \\
&x_j(t+\Delta t) = x_{j-1}(t+\Delta t) + h\, c_T \cos\theta_{j-1}(t+\Delta t) \\
&z_j(t+\Delta t) = z_{j-1}(t+\Delta t) + h\, c_T \sin\theta_{j-1}(t+\Delta t) \\
&S_j(t+\Delta t) = S_{j-1}(t+\Delta t) + h\, c_T
\end{aligned} \qquad (34)$$

Where

$$a_\tau = \tau_x\ddot{x}_{i,t} + \tau_z\ddot{z}_{i,t} = \cos\theta_{j-1}(t)\ddot{x}_{i,t} + \sin\theta_{j-1}(t)\ddot{z}_{i,t}$$

$$a_n = n_x\ddot{x}_{i,t} + n_z\ddot{z}_{i,t} = \sin\theta_{j-1}(t)\ddot{x}_{i,t} - \cos\theta_{j-1}(t)\ddot{z}_{i,t}$$

$$a_{1,j-1} = q\cos\theta_{j-1}(t) + \rho_c(\ddot{x}_{i,t}\sin\theta_{j-1}(t) + \ddot{z}_{i,t}\cos\theta_{j-1}(t));$$

$$a_{2,j-1} = q\sin\theta_{j-1}(t) - \rho_c(\ddot{x}_{i,t}\cos\theta_{j-1}(t) - \ddot{z}_{i,t}\sin\theta_{j-1}(t))$$

$$e_{1,j-1} = F_\tau + \lambda_\tau a_\tau \;;\; e_{2,j-1} = F_n + \lambda_n a_n \;;\; c_T = 1 + \frac{T_{j-1}(t+\Delta t)}{EA}$$

After determining the dynamic configuration, one can find the velocities and the accelerations:

$$\begin{aligned}
&\dot{x}_j(t+\Delta t) = \dot{x}_j(t) + \frac{x_j(t+\Delta t) - x_j(t)}{\Delta t};\; \dot{z}_j(t+\Delta t) = \dot{z}_j(t) + \frac{z_j(t+\Delta t) - z_j(t)}{\Delta t} \\
&\ddot{x}_j(t+\Delta t) = \ddot{x}_j(t) + \frac{\dot{x}_j(t+\Delta t) - \dot{x}_j(t)}{\Delta t};\; \ddot{z}_j(t+\Delta t) = \ddot{z}_j(t) + \frac{\dot{z}_j(t+\Delta t) - \dot{z}_j(t)}{\Delta t}
\end{aligned} \qquad (35')$$

j=1, 2, ……., n-1

Because the motion of the upper end - point A, (Figure 1) - is imposed, then the last connecting point for determining the velocity and the acceleration is the connecting point with the index n-1.

2.2- For the Cable Equilibrium Configuration in the Plane Xoz:

$$\frac{dT}{ds} = (q \sin\theta - F_\tau c_T); \quad \frac{d\theta}{ds} = (\frac{q}{T}\cos\theta + \frac{F_n}{T})c_T;$$

$$\frac{dx}{ds} = c_T \cos\theta; \quad \frac{dz}{ds} = c_T \sin\theta; \quad \frac{dS}{ds} = c_T \tag{36}$$

And one can obtain particular solutions for diverse cases, each case being characterized by specific boundary conditions.

2.1. Moored Systems

2.1.1 Static Analysis

The numerical computation for static analysis is described in the paper [12]. This analysis refers to the both cases, cable tangent to the bottom and cable detached from the bottom. For all the cases, we have a problem with boundary conditions at the both ends of cable (bi-local boundary condition).

2.1.2 DynamicAnalysis

The numerical computation begins from the equilibrium cable configuration as the initial mechanical cable state.

a. For the cable of a moored system (as shown in Figure1). The initial mechanical state of cable: initial cable configuration (the coordinates x, z of cable points in the coordinates system xOz), the tensions in all cable elements, the angles θ for all elements; the motion of the upper end (point A): x= $x_A(t)$, z= $z_A(t)$; the velocities and the accelerations of cable points. The cable is composed by a number of elements. For the elements that are placed on the sea bottom the tensions are the same and equal to the tension of the first element detached from the sea bottom.

b. The dynamic cable behavior is determined by solving the differential equations of cable motion (36). For determining the dynamic response to the excitation induced in the upper end the solution of the differential equations are determined on the steps of time, Δt. In the frame of each time step an iterative method is used for determining the cable dynamic configuration that must satisfy the boundary condition. These conditions can be attained by fitting the initial value of the cable tension, T_A, in the first element that is connected in the upper end A, and also by fitting the initial value of the angle θ_A of the same element.

The imposed boundary conditions are:

1. In the point a (upper end): $x_A = x_A(t)$, $z_A = z_A(t)$; the angle θ_A and the tension T_A are not known.
2. In the point D-for the cable tangent to the bottom, or the point O for the cable detached from the bottom-(lower end): z=0 is known.
3. The cable length is L.

The iterative process begin, first iteration, by fitting the value of the angle θ_A or of the tension T_A as follows:

$\theta_A = \chi_{k+1}\beta_t$, $T_A = \chi_{k+1} T_a$, where $1 \geq \chi > 0$; for the first iteration χ_1=1 and β_t=90^o, T_a -the admissible cable tension; for the second iteration, k=1, χ_1=0,5 and $\chi_{k-1} = 0$, k=1, χ_1=0,5 si $\chi_{k-1} = 0$, k=1,…, n_{max}, where n_{max} is the maximum number of iterations.

The computation stops if the boundary conditions are attained at the lower end, as follows:

At the time step κ-th:

$$x_{A,\kappa}(t+\Delta t) = x_A(t) + \Delta x_A(t+\Delta t) = x_A(t) + a_x(\sin\omega(t+\Delta t) - \sin\omega t) \text{ and}$$

$$z_{A,\kappa}(t+\Delta t) = z_A(t) + \Delta z_A(t+\Delta t) = z_A(t) + a_z(\sin\omega(t+\Delta t) - \sin\omega t);$$

Where the initial values of the coordinates of upper end A are the values that result for equilibrium configuration, $x_A(t_0) = x_A$, $z_A(t_0) = z_A$

If the point A is connected to a fixed body, for example a quay, and the motion of the moored vessel under the external excitations is known, $x_A = x_A(t)$, $z_A = z_A(t)$, the computation follows the same steps and the boundary condition at the lower-point A-are: $x_n = x_m$; $z_n = z_m$ where x_m and z_m are the imposed value of the coordinates of the point A .

2.2. Towed Systems

2.2.1Static Analysis

a. The method that was presented for a moored system is also applied for any towed system — floating body, cable or towing vessel. The coordinate system xOz is attached to the towed body in translational motion (Figure 3). In the cable's attachment point to the body, O, we have the following conditions: x = z = s = S = 0, □□□□. In the cable's attachment point to the towing ship (point A), we have one only stop condition: either the length of the cable, L, or another condition that depends on the engineering requirements. For example, should the stop condition be $x = d_0$, where d_0 is an imposed horizontal spacing between the body and the towing ship, or an imposed value of the depth of the towed body, $z= H_A$. In this case we need to take into account the velocity of the towing ship in the relationship (7a) that is now:

$$\mathbf{v_r}(z_i) = [[v_c(z_i)\sin\gamma] + v_x(z_i) + v_{TS}]\mathbf{i} + [[-v_c(z_i)\cos\gamma] + v_z(z_i)]\mathbf{k}$$

where v_{TS} is the velocity of the towing ship as is shown in Fig. 3 for the underwater towed vehicle.

The tension in the attachment point of the cable, T_o, is understated (this tension is due to the hydrodynamic forces acting on the body combined with the vehicles weight in water). Also, is known the angle θ_o.

The iterative process begin from the point O and the computation stops if the boundary conditions are attained at the upper end: if $z= H_A$. The another boundary conditions for stops computation could be: if $x= d_0$ or if s=L

In these cases, we have a problem with boundary conditions at only one of the cable ends.

2.2.2 Dynamic Analysis

A complete dynamic analysis takes into account the motion of all three components of the system: towed vessel, cable and towing vessel. It is a complex problem that not is our preoccupation in this paper.

However, it must underline that our proposed model for cable dynamics can be used for all cases above presented.

3.The Compound Marine Cables

A typical compound marine cable is made of several components, and one can use the mathematical model that was established for single cable for any compound marine cables (Figure 4). However, the conditions of continuity of the compound marine cables must be specified since the boundary conditions are unknown at the points where the components are located. Moreover, the conditions of continuity in the joints between the elements must be satisfied. The starting point for the numerical computation can be any one of both compound marine cables ends and must be re-started at each joint. The static analysis is described in the paper [12].

The dynamic behavior of the cable is determined by solving the differential equations of motion *i.e.*, Eq. (35). For determining the dynamic response to the excitation induced in the upper end, the solution of the differential equations is determined along the time, considering the time step amplitude Δt, and an iterative method is used at each instant for determining the riser configuration that satisfies the boundary conditions. These conditions can be attained by fitting the initial values of the cable tension, T_o, and of the angle θ_o in the element that is located at the lower end O.

The imposed boundary conditions are:

- At point O (lower end): x=z=0; the angle θ_o and the tension T_o are unknown.

- At point A (upper end): $x= x_A(t)$, and $z= z_A(t)$.

- The buoyancy, B, and length of both parts of the cable, L_1, L_2 are also imposed.

The iterative process begins by fitting the value of the angle θ_o or of the tension T_o as follows:

- for the first iteration: $\theta_o = \chi_k\beta_t$, $T_o = \chi_k\ T_a$, where $1 \geq \chi > 0$; χ_1=1 and β_t =90^o

- for the second iteration: $\theta_o = \chi_k\beta_t$, $T_o = \chi_{k+1}\ T_a$, k=1, χ_1=0,5 si $\chi_{k-1} = 0$, k=1,…, n_{max}.

The computation stops if the boundary conditions are attained at the upper end, as follows:

At the κ-th time step:

For Section 1 (between the point O and point O_B):

- Position of point A at the time step κ-th:

$$x_{A,\kappa}(t+\Delta t) = x_A(t) + \Delta x_A(t+\Delta t) = x_A(t) + a_x(\sin\omega(t+\Delta t) - \sin\omega t) \text{ and}$$

$$z_{A,\kappa}(t+\Delta t) = z_A(t) + \Delta z_A(t+\Delta t) = z_A(t) + a_z(\sin\omega(t+\Delta t) - \sin\omega t);$$

where the initial values of the coordinates of upper end A are the values that result for equilibrium configuration, $x_A(t_0) = x_w$, $z_A(t_0) = H_A$

- The cable motion is computed by using an iterative method for solving Eq. (35), where the tangential and normal components of the hydrodynamic force per unit length of the cable are determined through Eq. (34).

The conditions for re-starting (Figure 5) the computation are: if s= L_1, then

$$\begin{cases} A_{xf} = D_f h_f \;\; ; \; FH_x = \dfrac{1}{2} A_{xf}\, \dot{x}(z_\mu) \left|\dot{x}(z_\mu)\right| \; ; \; \lambda_{fn} = \dfrac{1}{2}\rho_w c_{inf} \pi D_f^2 \; ; \; \lambda_{f\tau} = \dfrac{1}{2}\rho_w \pi D_f^2 \\ \theta_{\mu+1} = \operatorname{arctg} \dfrac{T_\mu \sin\theta_\mu + (m_f + \lambda_{fn})\ddot{x}_\mu + FH_x}{B + (m_f + \lambda_{f\tau})\ddot{z}_\mu + T_\mu \cos\theta_\mu} \\ T_{\mu+1} = \dfrac{B + (m_f + \lambda_{f\tau})\ddot{z}_\mu + T_\mu \cos\theta_\mu}{\cos\theta_{\mu+1}} = \dfrac{T_\mu \sin\theta_\mu + (m_f + \lambda_{fn})\ddot{x}_\mu + FH_x}{\sin\theta_{\mu+1}} \end{cases} \quad (37)$$

Where m_f is the mass of the buoy, and $\lambda_{f\tau}$ and λ_{fn} are the tangential and normal added mass of the buoy.

For Section 2 (from point O_B to point A - upper end):

The numerical computation begins at the re-starting point, O_B (see Figure 6).

The following boundary conditions are used

$$s = L_1 \, ; \; \theta = \theta_{\mu+1} - \pi/2; \; T = T_{\mu+1} \quad (38)$$

and the computation stops if s= $L_1 + L_2$ and x= $x_{n,\kappa}(t+\Delta t)$, z= $z_{n,\kappa}(t+\Delta t)$.

If the number of iterations, n, becomes equal to the maximum number n_{max} and the stop computation condition is not accomplished, then the solution is not valid. In this case, the boundary conditions must be revised.

For the integration of the Eq. (32) and Eq. (33) we have used the following optimizing procedure:

a) Valid solutions

The solution that corresponds to the pair $(T(1), \theta(1))$ is valid if it satisfies all the conditions but $x(n) = x_w$ and z(n) = H_A. The errors (or conditions) that make a solution not valid are the following:

1. z(j) > H_A at Section 1;
2. (j) < 0º at Section 1;
3. (j) > 90º at Section 1;
4. $_{buoy}$ > 0, where □$_{buoy}$ is the angle of the cable just after the buoy (at the right side of the buoy);
5. (j) > 0º ;
6. z(j) < 0 at Section 2;
7. (j) > 90º at Section 2;
8. (j) < □(j-1) at Section 2.

The variables were scalled as follows: $T(1) = \overline{T}.T_{ad}$, $\theta(1) = \overline{\theta}.90$ (39)

Where

$$\overline{T} \in [0,1], \overline{\theta} \in [0,1] \quad (40)$$

b) The system of equations

The next step for the admissible solutions $(T(1), \theta(1))$ or $(\overline{T}, \overline{\theta})$ is to solve the system of equations

$$\{x(n) = x_w;\ \ z(n) = H_A \tag{41}$$

that is expressed in the scaled form through

$$f_1(\overline{T}, \overline{\theta}) = \frac{x(n) - x_w}{x_w} = 0\ ;\ f_2(\overline{T}, \overline{\theta}) = \frac{z(n) - H_A}{H_A} \tag{42}$$

and the following four inequations must be added:

$$0 \le \overline{T} \le 1;\ \ 0 \le \overline{\theta} \le 1 \tag{43}$$

It is very difficult to use a standard algorithm to solve these in equations. Hence, the problem is reformulated as an optimization problem with simple marginal restrictions. This means that the following objective function should be minimized:

$$g = \frac{1}{2}(f_1^2 + f_2^2) \tag{44}$$

By taking into consideration the restrictions of simple margin expressed by Eq. (43)

A few methods can be used for solving the optimization problem:

- A version of the quasi-Newton (Broyden-Fletcher-Goldfarb-Shanno, BFGS) algorithm especially adapted to consider the restrictions. The gradient of the function, g, is computed through finite differences.
- A version of the Newton algorithm where the Hessian matrix is aproximated by finite differences, but modified in order to consider the restrictions. Additionally, the finite differences approximation of the first order derivatives is attained with a higher order of precision.
- A Levenberg-Marquardt least squares technique where the Jacobian matrix is approximated through finite differences.

The aforementioned algorithms were used to solve the non-linear problem represented by Eq. (44) with several different starting points, always leading to the same solution. This proves that the solution for Eq. (44) is unique.
The method for determining the dynamic behaviour of the cable can be summarized as follows:

i. The initial configuration of the cable - corresponding to the equilibrium configuration – is determined according to the method that is described in reference [12] and detailed above for different cases. Eq. (33) is used for analyzing the bi-dimensional configurations that are discussed in the present paper.

ii. The cable motion is initiated by applying an external excitation, and the starting point is the static equilibrium configuration. In most cases the excitation is defined as the displacement, the velocity and the acceleration of the upper end. The dynamic response of the cable is attained by integration of Eq. (32), and one should notice that this is done through a computational procedure that is similar to the one that is used for determining the equilibrium configuration of the cable. This is due to the fact that Eq. (32) is of the same kind of Eq. (33), which relates to the dynamic equilibrium equation, in which the boundary and the continuity conditions must be satisfied at each instant of the cable motion.

4.Computed Results

Case Studies

An original computer program (DYNCAB) based on the above-mentioned mathematical model was written to implement the numerical computation of the complete analysis - static equilibrium configuration and dynamic behavior of marine cables. An analysis of the static equilibrium configuration by using the abovementioned program, as well as the validity of the approach, was made by Matulea *et al.* [12]. This was done by comparing the DYNCAB results with the results obtained through a lamped mass method [13]. Additionally, Matulea *et al.* [12] have shown that there is a good correlation between the result of numerical simulations and the results attained with the tank tests. The present paper shows that the approach is also suitable for a comprehensive dynamic analysis in order to find the best solution for different operating conditions.

Case study: Model of a compound cable with a float

Now, our goal is to validate the proposed approach for the dynamic behaviour of a marine cable.

Figure 8a and 8b show the results obtained by using the proposed method. For a matter of comparison, the results obtained through the "lamped mass method" of Nakajima *et.al* [13] are also shown. As one can see, there is a good correlation between the proposed method and the "lamped-mass method".

The values of the vertical component of the cable tension in the upper end A, $T_{A,z}$, are the same for both the proposed method and "lumped-mass method". The same apply to the experimental tests as well (see Figure 8a and 8b). In the Figure 8a there are noted by $T^0_{A,x}$ and $T^0_{A,z}$ the initial values of the tension components on coordinate axes in the upper end A. These values result from static analysis.

5. Concluding Remarks

A novel method based on a very flexible and efficient mathematical model for the complete static and dynamic analysis of marine cable dynamics was presented in this paper. The model takes into account most of the non-linearities of marine cable, such as nonlinear water velocity profiles, large marine cable deflections and nonlinear constraints.

The computer program that implements the method is a general purpose one. It can handle the instability that is due to bifurcation, and makes it possible to study a large number of operating conditions and system configurations in a cost-effective way.

Using the program for analyzing some cases of marine cable tested the soundness of the model. The numerical experiments that are presented in the paper show a very good correlation with the results obtained by Nakajima *et al.* [13] as shown in Figures 7a and 7b.

The use of lumped mass models imparts some practical difficulties: first, the marine cable length must be provided as input data; second, the data preparation is a laborious task; third, it is hard to find out the location of the point where the marine cable contacts with the seabed.

On the contrary, the mathematical model hereby presented proved to be more flexible and efficient because it is based in non-restrictive assumptions that express very well the most typical engineering design requirements. In fact, the new model allows for the use of the cable arch as an integration variable, as well as the marine cable coordinates. In addition, the proposed model makes very easy to consider different boundary conditions as well as distinct continuity conditions.

As a general conclusion arising – both from the theoretical standpoint and the results of the performed calculations - one can point out:

- static analysis is particularly useful in defining the design parameters and the initial configuration that is required to perform the dynamic analysis;
- dynamic analysis is required to evaluate the occurrence of large stresses due to dynamic loading, and the analysis in the time domain allows a very complete and accurate prediction of all the most important aspects of the marine cables behavior.

In order to improve the computer program, it is still necessary to add the possibility of taking into account the variable marine cable bending stiffness and the variable buoyancy close to the sea surface, as a means to allow the analysis of some more involved marine cables systems that are being studied by some of the program users.

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Nomenclature

- $\square_t$ - virtual angle used in computation process that must be found by trial and error.
- θ - angle between the tangent to the riser and the z-axis at the current point
- γ - angle between the unit vector $\boldsymbol{\tau}$ and the vector of the current water velocity at coordinate z
- $v(z)$ - relative velocity to the water
- $v_c(z)$ - water current velocity
- ρ_w - water density
- (r_v) - vector of the current position, at the time t, of the point M
- $(\dot{r}_{v,t})$ - velocity vector of the point M at the time t
- $(\ddot{r}_{v,t})$ - acceleration vector of the point M at the time t
- F_n, F_t - normal and tangential components of hydrodynamic forces acting on the unit cable length
- (F_λ) - added mass vector
- λ_τ, λ_n - added mass for a unit length of cable
- $c_{in}, c_{i\tau}$ - inertial coefficients
- A - effective area of the cable section
- c_n, c_τ - coefficients of hydrodynamic forces

- d_c - characteristic diameter of the riser
- E - longitudinal modulus of elasticity
- [R] - transform matrix between the system of vectors □, **n** and the coordinate system xOz
- h - computational step in the finite difference method
- H - water depth
- q - line weight in water per unit length
- s - arc of non-stretched line at current point
- S - arc of the stretched line at current point
- T - cable tension in the current point
- x, z - coordinates of the current point
- B - buoyancy
- L_1, L_2 - lengths of the components of a multicomponent line
- x_A, z_A - coordinates of the upper end

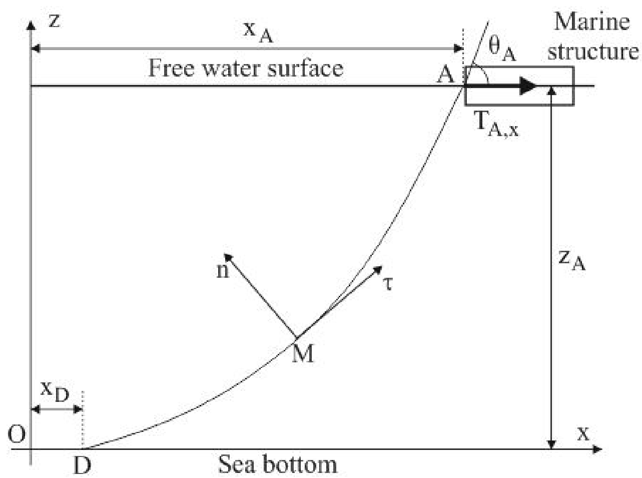


Figure 1 Mooring Cable

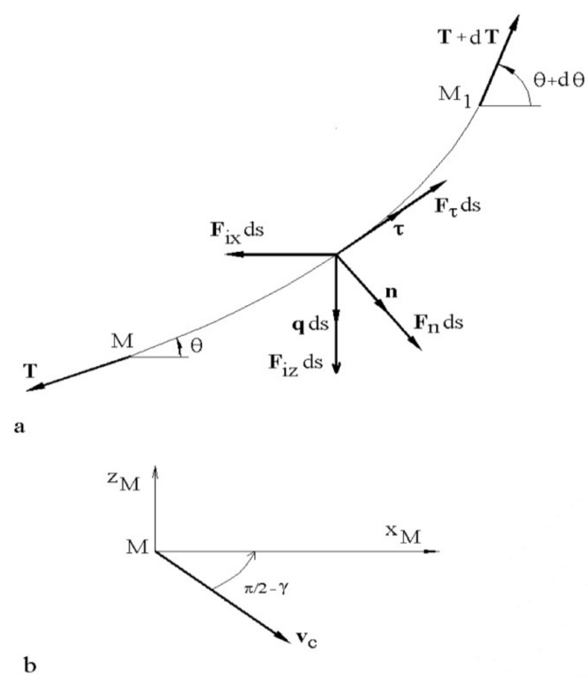


Figure 2. The Forces Acting on an Element of the Cable

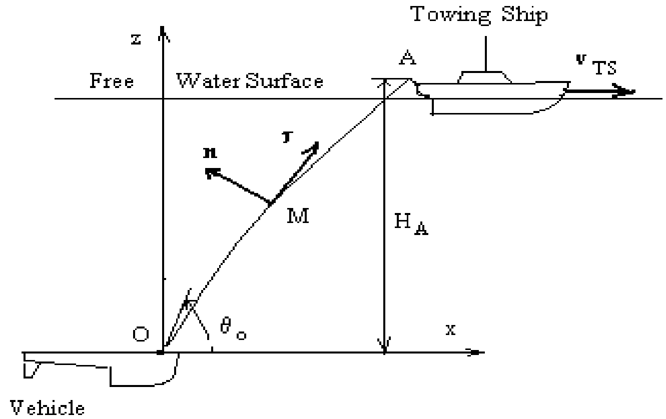


Figure 3 Towing Cable

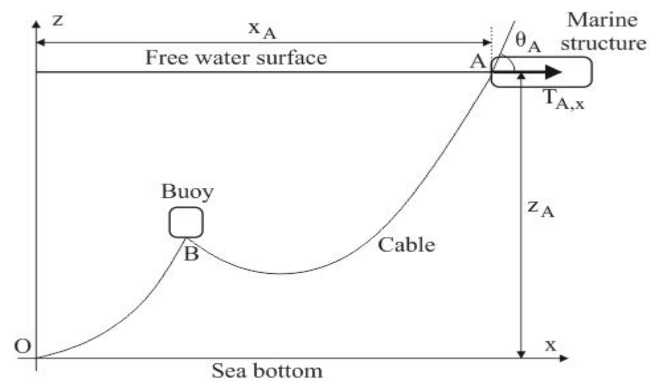


Figure 4 Mooring Cable with a Float

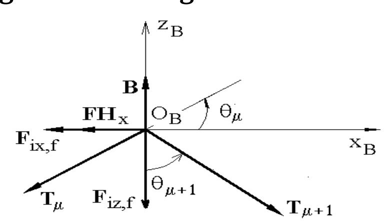


Figure 5. The dynamic Loading of the Buoy

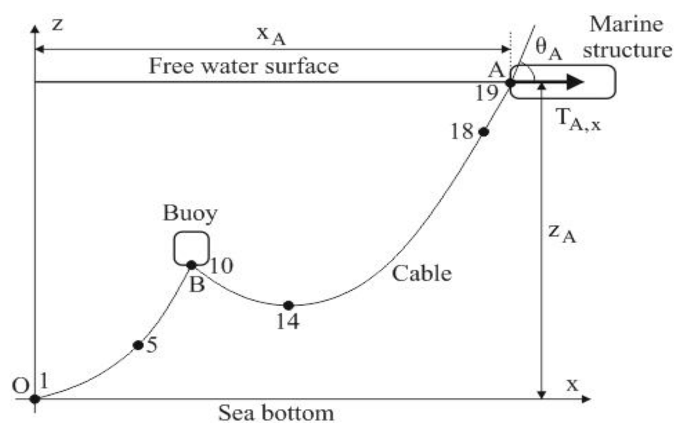


Figure 6 Case study: Cable with a Buoy

Table 1-Cable Characteristic

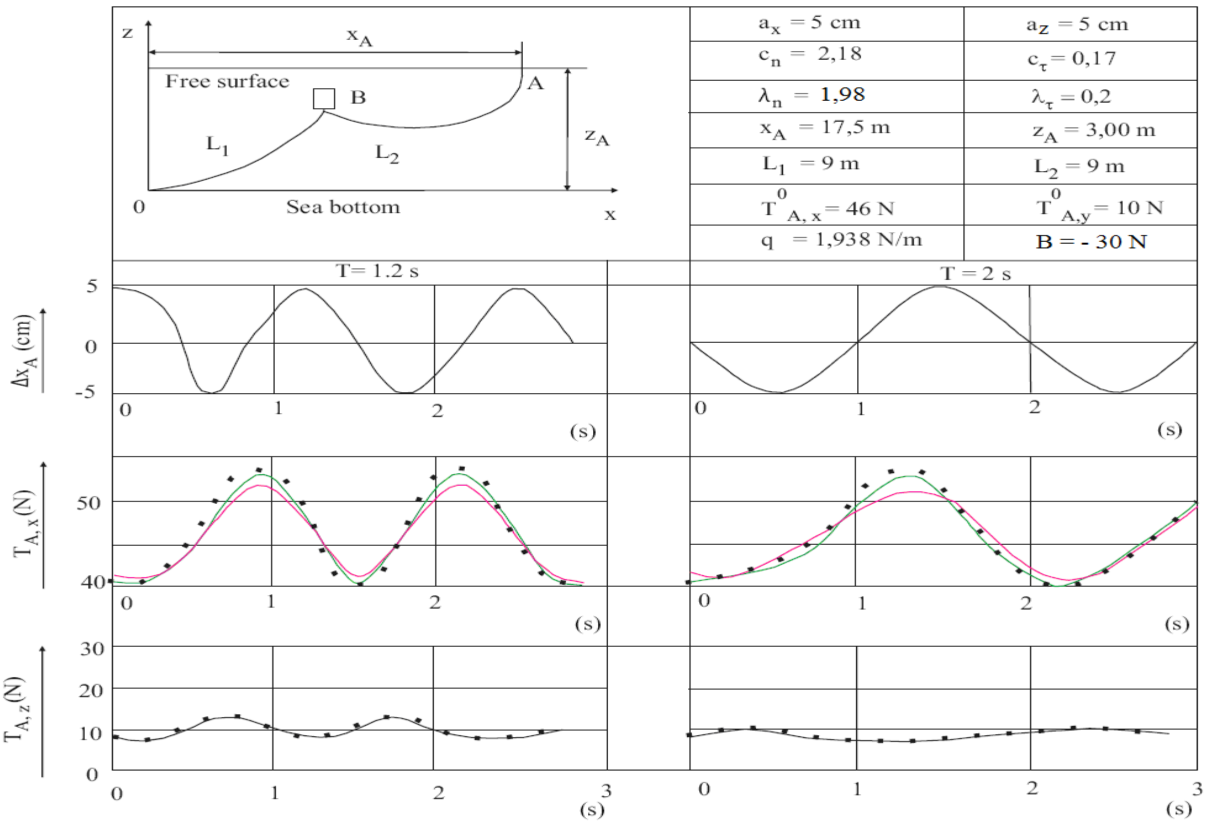
Material	-	Steel
Weight in air	N/m	2,22
Weight in water	N/m	1,938
Volume/unit length	cm^3/m	28,2
Elasticity modulus (E)	N/m^2	$21,5x10^{10}$
Diameter d	cm	0,599

Table 2-Buoy

Weight in air	0,7 N
Weight in water	-30 N

Table 3-Characteristic of Equilibrium Configuration

1	0,00	46,2	0,00	0,00	0,00
5	4,53	49,5	0,00	4,11	0,9
10	9,00	47,4	15,30	9,04	2,35
14	13,10	47,5	20,25	13,02	2,39
18	16,10	48,2	22,34	16,52	2,73
19	18,00	50,35	24,00	17,60	3,00



c_n, c_τ, λ_n and λ_τ are determinated by experimental tests

The results of numerical computation obtained with the proposed method

The results of experimental testing - [13]

The results of numerical computation obtained with the "lumped mass method - [13]

Figure 7a: The Model Behavior under Dynamic Loading

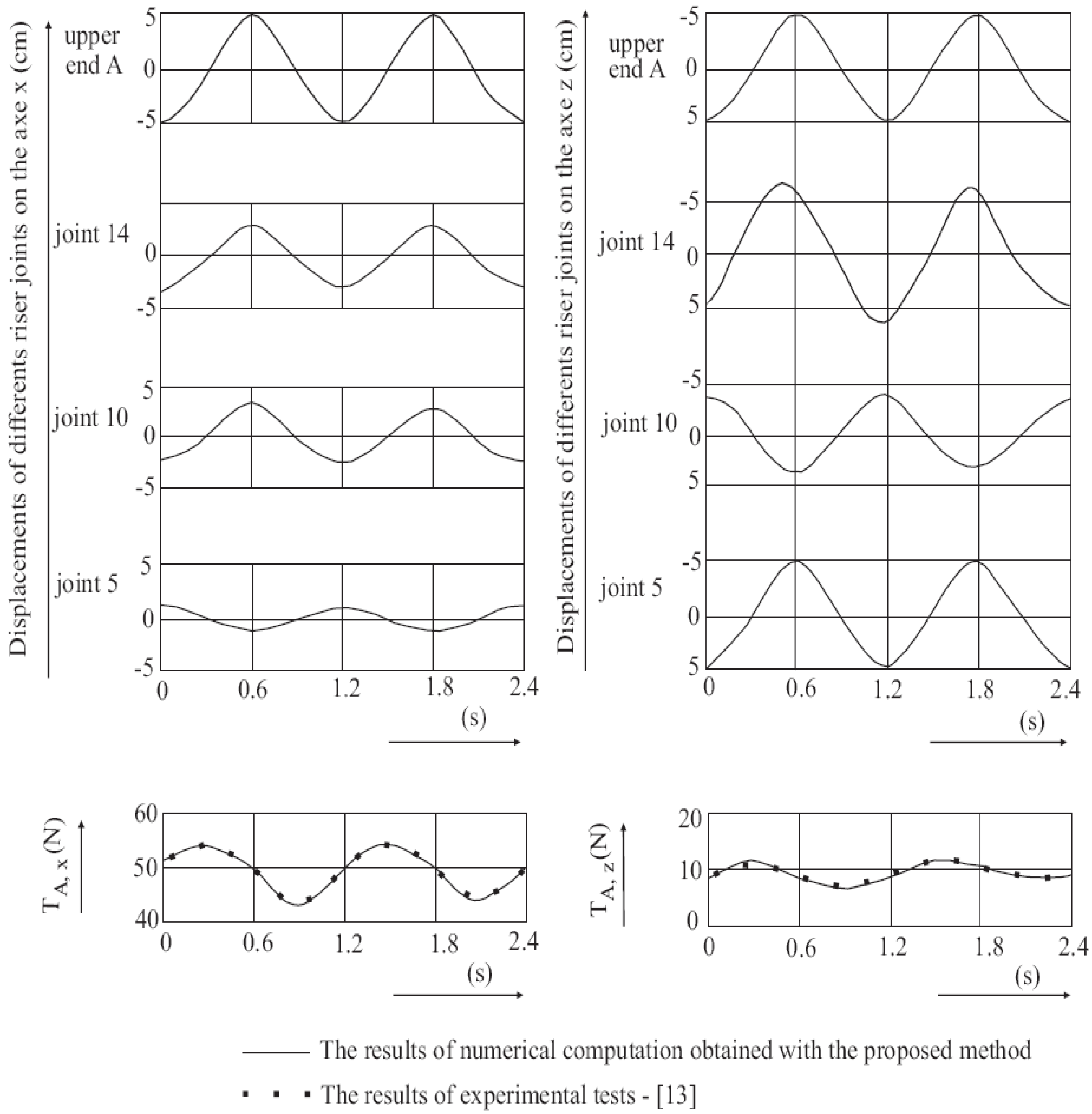


Figure 7b: The Model Behavior under Dynamic Loading for the Time Period T=1.2 S