On the Antenna Gain Formula

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Abstract

Two formulas were derived for estimation of antenna gain. The first formula is derived based on the method of gain in the field intensity of the array antenna. The second is derived based on the existing gain-comparison method. The two formulas were used to compute the gain of a two rectangular folded array antenna, as an example, and using the $\lambda/2$ antenna as the reference isotropic antenna. The computation results were compared with the experimental measurements. The normalized root-mean-square errors of 0.52% and 0.3% were obtained with the first and second formulas respectively. The two methods can be used to determine the gain of antennas.

Keywords: Antenna, Gain, Theoretical, Formula

1. Introduction

There are several parameters for characterization and analysis of the antenna performance namely radiation pattern, gains, directivity, beam width, voltage standing wave ratio, polarization and bandwidth to mention few. Although, they are all related, gain is the most important figure-of-merit that describes the performance of the antenna. (Kraus, 1988; Balanis, 2005). The gain G of an antenna is defined as the ratio of the power density in the direction of maximum radiation to that which will be observed at the same distance over an isotopic antenna while radiating the same total power, and typically it is expressed in the unit of dBi (Kraus, 1988; Balanis, 2005). It is not an overstatement that high performance communication system design requires accurate estimation of antenna gain. It is sometimes almost impossible or very difficult to measure the antenna gain because of its relation to the antenna structure and frequency of the radio wave.

Various experimental measurement methods of antenna gain determination namely, the absolute-gain and gain-comparison using near-field and far field anechoic chamber, non anechoic and open air far-field techniques have been employed by radio scientist and engineer over the years (Collin, 1985; Kraus, 1988; Vishal, 2000; Kai, 2000; Kraus et al, 2002; Balanis, 2005). These methods are subjected to errors due to the variation in system's frequency, the far-field criteria, the antennas alignment for bore sight radiation, impedance and polarization perfect match conditions, and proximity effects and multipath interference. Recently, some efforts have been made in improving the antenna measurement techniques (Laitinen et al, 2006; Dolecek and Schejbal, 2009; Loredo et al, 2009; Gregson and Hindman, 2009). The experimental measurement method though very accurate for the antenna gain estimation if done properly, they are very rigorous, cumbersome and expensive. To the best of my knowledge based on the available open literatures, despite many challenges of experimental measurement method, there is no reliable and high accurate and self sufficient theoretical mathematical formula without experimental measurement that can be employed to determine antenna gain.

This paper reviews the existing antenna gain estimation methods and presents other new methods (theoretical formulas) for estimating the gains of antennas. As an example, experimental results of the of a two rectangular folded array antenna are presented and compared with the numerical computation of the antenna gain using the new formulas.

2. Review of the existing antenna gain formulas and measurement techniques

The relationship between the gain G and directivity D that describes a transmitting antenna which allows most of the transmitted power to be sent in the wanted direction is given as (Collin, 1985; Kraus, 1988; Kai, 2000; Balanis, 2005):

$$G = kD, \qquad (1)$$

Where, k is the aperture efficiency of the antenna to radiate the energy presented to its terminals, and it is given by the expression:

$$k = \frac{\max imum \text{ effective area of the antenna } (\mathrm{A_{em}})}{physical \text{ area of the antenna } (\mathrm{A_p})}, \qquad (2)$$

$(A_{em} \le A_p \,, 0 \le \mathrm{k} \le 1)$

Also, $$k = \frac{Radiation \text{ resistance } (\mathrm{R_{rad}})}{Radiation \text{ resistance} + \text{ohmic loss } (\mathrm{R_L})}, \qquad (3)$$

$(R_L <<< \mathrm{R_{rad}})$

While, the directivity, D(θ, ϕ) of the antenna is the ratio of the Poynting power density S(θ, ϕ) to the power density radiated by an isotropic source written as (Kai, 2000): $D(\theta,\phi) = \dfrac{\max imum \left|\mathrm{S}(\theta,\phi)\right|}{P_t / 4\pi r^2}$, (4)

Where,

$$\left|S(\theta,\phi)\right| = \frac{1}{2}\mathrm{Re}(\vec{E} \times \vec{H}) = \frac{1}{2}\left|E\right|^2, \qquad (5)$$

The gains of the transmitting and receiving antennas separated by a distance $r > \dfrac{2A_p^2}{\lambda}$ can be related by the Friis transmission formula which is given as:

$$\mathrm{P_r} = \frac{\mathrm{P_t} G_t A_{er}}{4\pi r^2}, \qquad (6)$$

Where, A_{er} is the effective area of the receiving antenna which is given as:

$$A_{er} = \frac{G_r \lambda^2}{4\pi}, \qquad (7)$$

Therefore, Equation (6) can be written as:

$$\frac{P_r}{p_t} = \left(\frac{\lambda}{4\pi r}\right)^2 G_t G_r, \qquad (8)$$

Where, λ is the operating wavelength in meters.

One of the antenna gain estimation techniques is called absolute-gain, and it is based on Friis transmission formula. Under this technique, there are four ways namely; two-antenna method (2AM), Three-antenna Method (3AM) and other two: extrapolation method (EM) and Grand Reflection Range Method (GRRM) which depend on the 2AM and 3AM (Vishal, 2000; Balanis, 2005). 2AM is employed for two identical antennas, (transmitting and receiving antennas), where Friis transmission formula is written in a logarithmic decibel form as:

$$G_t + G_r = 20\log_{10}\left(\frac{4\pi r}{\lambda}\right) + 10\log_{10}\left(\frac{P_r}{P_t}\right), \qquad (9)$$

Where, G_t = gain of the transmitting antenna in *dB*, G_r = gain of the receiving antenna *dB*,
P_r = received power in Watt, P_t = Transmitted power in Watt, r = distance between transmitting and receiving antennas in meters.
Since transmitting and receiving antennas are identical, Equation 8 becomes:

$$G_t = G_r = \frac{1}{2}\left[20\log_{10}\left(\frac{4\pi r}{\lambda}\right) + 10\log_{10}\left(\frac{P_r}{P_t}\right)\right], \qquad (10)$$

Any of the parameter: r, λ, P_r and P_t can be measured by near-field and far field anechoic chamber, non anechoic and open air far-field techniques, and the gain is computed.

The 3AM is employed when the transmitting and receiving antennas are not identical. Transmitted and received powers at the antenna terminals are measured between three arbitrary antennas (a, b, c) at a known fixed distance. The (Friis transmission formula) written in *dB*, is used to develop three equations and three unknowns as (Vishal, 2000; Balanis, 2005):

(a - b combination): $$G_a + G_b = 20\log_{10}\left(\frac{4\pi r}{\lambda}\right) + 10\log_{10}\left(\frac{P_{rb}}{P_{ta}}\right), \qquad (11)$$

(a - c combination): $$G_a + G_c = 20\log_{10}\left(\frac{4\pi r}{\lambda}\right) + 10\log_{10}\left(\frac{P_{rc}}{P_{ta}}\right), \qquad (12)$$

(b - c combination): $$G_b + G_c = 20\log_{10}\left(\frac{4\pi r}{\lambda}\right) + 10\log_{10}\left(\frac{P_{rc}}{P_{tb}}\right), \qquad (13)$$

The three equations (11, 12, and 13) are solved to determine the unknown gains G_a, G_b, G_c provided r, λ, P_{rb}, P_{rc}, P_{ta}, P_{tb} are known.

The second technique of antenna gain estimation is called gain-comparison or gain transfer techniques (GCT or GTT). The antenna gain is measured by comparing the antenna under test, AUT against a known standard antenna, S gain (Kraus, 1988; Vishal, 2000; Balanis, 2005; Dolecek and Shejbal, 2009). At lower frequencies (< 1GHz) half-wave dipole antenna is employed as the standard and at higher frequencies (>1GHz), a high gain directional horn antenna is employed as the standard. The procedure requires two sets of measurements. In the first, the test antenna is used as the receiving antenna and the received power $P_{r(AUT)}$ is measured and recorded. In the second set, the test antenna is replaced by the standard gain antenna and the received power $P_{r(S)}$ is measured and recorded.

The gain G_{AUT} of the antenna under test is then deduced from the expression given by (Balanis, 2005):

$$G_{AUT} = G_S + 10\log_{10}\left(\frac{P_{r(AUT)}}{P_{r(S)}}\right), \qquad (14)$$

Other efforts made in area of the antenna measurements are listed as follows:

Laitinen et al, 2006 examined iterative probe-correction technique as a possible method to correct errors associated with probes from the manufacturer. It is because these errors usually lead to inaccurate antenna measurements.

Dolecek and Schejbal, 2009 verified a simple existing formula for estimation of gain of the shaped-beam antenna. The results of the experimental measured gain for five Hogg-horn antennas as examples were comparable with the formula:

$$G = k\frac{d_b}{\phi_1\phi_2}, \qquad (15)$$

Where ϕ_1 and ϕ_2 are the half-power beamwidths in the principal planes, and d_b is a constant (the directivity-beamwidth product, the values of 26,000 up to 52,525 are possible).

Loredo *et al*, 2009 presented a technique to eliminate the undesired contributions causing errors in the antenna measurement. In the paper titled measurement of low gain antenna in non anechoic test sites through wide band channel characterization and echo cancellation. The measurement techniques based on the Fourier Transform algorithm starts from measurement of the antenna responses in the frequency domain, followed by Fourier Transformation of data to the time domain, detection and gating of the undesired echo in the time domain, back to the frequency domain, and retrieval of the antenna radiation patterns (Characteristics) at the frequency of interest (4 GHz – 12 GHz). Seven different standard pyramidal-horn antennas and a monopole antenna were used AUTs and identical Pyramidal-horn antennas were used as probes. Although the technique evaluated under different conditions demonstrated fairly high accuracy, and even can be adopted for antenna measurement in an anechoic chamber, it is very costly experimentally and computationally.

3. Derivation of the new formulas

Method 1: Array antenna gain

Consider a standard array of two rectangular folded antenna side by side as shown in the figure 1. Each element of circumference C is carrying current I. The far-field electric and magnetic strength of one element of the array in the horizontal plane are respectively given by (Kraus, 1988; Kraus et al, 2002; Balanis, 2005):

$$E(\phi)=\frac{60\pi C_\lambda I}{2r}J_1(C_\lambda \sin\phi),\quad \mathrm{Vm}^{-1} \qquad (16)$$

$$H(\phi)=\frac{C_\lambda I}{2r}J_1(C_\lambda \sin\phi),\quad \mathrm{Am}^{-1} \qquad (17)$$

Where,

$$C_\lambda=\frac{\mathit{Circumference}\text{ of the loop antenna, C}}{\text{Wavelength, }\lambda\text{ of the radio wave radiated}}, \qquad (18)$$

ϕ is the azimuth angle in the horizontal plane about the vertical plane, and J_1 is the first-order Bessel function given as:

$$J_1(C_\lambda \sin\phi)=\frac{C_\lambda \sin\phi}{2}\left[1-\frac{1}{1!2!}\left(\frac{C_\lambda \sin\phi}{2}\right)^2+\frac{1}{2!3!}\left(\frac{C_\lambda \sin\phi}{2}\right)^4+\ldots\ldots\ldots\right], \qquad (19)$$

Here, the Bessel function is approximated to 3rd term because the dimensions (length and breadth) of the rectangular loop in this work is less than the wavelength o the radio frequency signal (100 – 650MHz) considered. As the power exponents increase the corresponding finite term become insignificant.

Let us concentrate on the electric field only. The field intensity E_1 due to the single element 1 of the array can be expressed as:

$$E_1\ \alpha\ \mathrm{E}(\phi)\Rightarrow \mathrm{E}_1=\varphi E(\phi), \qquad (20)$$

Where, φ is a dimensionless constant of proportionality.

Based on the principle of pattern multiplication, the total field intensity E_T due to the 2 elements in the array can be expressed as:

$$\mathrm{E_T}=E_1+E_2=2E_1, \qquad (21)$$

$$E_T(\phi)=\frac{60\pi\varphi C_\lambda I}{r}\left[\frac{C_\lambda \sin\phi}{2}-\frac{1}{1!2!}\left(\frac{C_\lambda \sin\phi}{2}\right)^3+\frac{1}{2!3!}\left(\frac{C_\lambda \sin\phi}{2}\right)^5\right], \qquad (22)$$

$$E_T(\phi)=\frac{60\pi\varphi C_\lambda I}{r}\left[\frac{C_\lambda \sin\phi}{2}-\frac{1}{16}C_\lambda^3\left(\frac{3\sin\phi-\sin 3\phi}{4}\right)+\frac{1}{384}C_\lambda^5\left(\frac{\sin 5\phi-5\sin 3\phi+10\sin\phi}{16}\right)\right], \qquad (23)$$

$$E_T(\phi)=\frac{60\pi\varphi I}{r}\left[\left(\frac{C_\lambda^2}{2}-\frac{3C_\lambda^4}{64}+\frac{10C_\lambda^6}{5824}\right)\sin\phi+\left(\frac{C_\lambda^4}{64}-\frac{5C_\lambda^6}{5824}\right)\sin 3\phi+\frac{C_\lambda^6}{5824}\sin 5\phi\right], \qquad (24)$$

$$Let\ \left\{\begin{array}{l} \mathrm{a}=\dfrac{\mathrm{C}_\lambda^2}{2}-\dfrac{3C_\lambda^4}{64}+\dfrac{10C_\lambda^6}{5824}=\dfrac{0.5832}{\lambda^2}-\dfrac{0.063773}{\lambda^4}+\dfrac{0.002725}{\lambda^6} \\ b=\dfrac{C_\lambda^4}{64}-\dfrac{5C_\lambda^6}{5824}=\dfrac{0.021258}{\lambda^4}-\dfrac{0.001362}{\lambda^6} \\ c=\dfrac{C_\lambda^6}{5824}=\dfrac{0.000272}{\lambda^6} \end{array}\right\}, \qquad (25)$$

Therefore,

$$E_T(\phi) = \frac{60\pi\varphi I}{r}(a\sin\phi + b\sin 3\phi + c\sin 5\phi), \qquad (26)$$

Assuming the total power into the array is P, and no heat losses (i.e. $A_e = A_p$):
Power in the element 1 is:

$$P_1 = I_1^2(R_{11} + R_{12}), \qquad (27)$$

Power in the element 2 is:

$$P_2 = I_2^2(R_{22} + R_{21}), \qquad (28)$$

But, $R_{11} = R_{22}$; $R_{12} = R_{21}$; and $I_1 = I_2 = I_{rms}$.

Where, R_{11} and R_{22} are the self (input) resistances of the elements 1 and 2 respectively; R_{12} and R_{21} are the mutual resistances due to the elements 2 and 1 respectively.

Therefore, the total power to the array is;

$$P = P_1 + P_2 = 2I_1^2(R_{11} + R_{12}), \qquad (29)$$

Equation 29 yields:

$$I = \sqrt{\frac{P}{2(R_{11} + R_{12})}}, \qquad (30)$$

Substituting Equation (30) into (26) yields;

$$E_T(\phi) = \frac{60\pi\varphi}{r}\sqrt{\frac{P}{2(R_{11} + R_{12})}}(a\sin\phi + b\sin 3\phi + c\sin 5\phi), \qquad (31)$$

Using λ/2 (half-wave dipole) antenna as the reference isotropic antenna, assuming no heat losses, the current I_o at its terminals under the same power supply P is:

$$I_o = \sqrt{\frac{P}{R_{oo}}}, \qquad (32)$$

Where, R_{oo} is the self-resistance of the isotropic reference antenna.
The electric far-field intensity due to this isotropic reference antenna is given as (Kraus, 1988):

$$E_{\lambda/2}(\phi) = \frac{R_{oo}\varphi I_o}{r}\,\frac{\cos\left(\frac{\pi}{2}\cos\phi\right)}{\sin\phi}, \qquad (33)$$

Substituting Equation (32) into (33) yields:

$$E_{\lambda/2}(\phi) = \frac{R_{oo}\varphi}{r}\sqrt{\frac{P}{R_{oo}}}\,\frac{\cos\left(\frac{\pi}{2}\cos\phi\right)}{\sin\phi}, \qquad (34)$$

The gain in field intensity, *GFI*, of the two rectangular folded array antenna in this study according to Kraus 1988 is given as:

$$GFI = \frac{E_T(\phi)}{E_{\lambda/2}(\phi)}, \qquad (35)$$

Therefore,

$$GFI = 60\pi\sqrt{\frac{1}{2R_{oo}(R_{11} + R_{12})}}\,\frac{\left(\frac{a}{2} - \frac{a}{2}\cos 2\phi + b\sin\phi\sin 3\phi + c\sin\phi\sin 5\phi\right)}{\cos\left(\frac{\pi}{2}\cos\phi\right)}, \qquad (35)$$

Where,

$$R_{12} = 30\left\{2\cosh(\beta d) - \cosh[\beta(\sqrt{d^2 + L^2} + L)] - \cosh[\beta(\sqrt{d^2 + L^2} - L)]\right\}, \qquad (36)$$

$$\beta = \frac{2\pi}{\lambda},$$

Method 2: General antenna gain

Applying the Friis transmission formula (Equation 6) to the gain-comparison formula (Equation 14) yields gain formula which can be used for all antennas and is given as:

$$G_{AUT} = G_S + 10\log_{10}\left(\frac{A_{er(AUT)}}{A_{er(S)}}\right), \qquad (37)$$

4. *Results and Discussion*

In this paper two formulas were derived for estimation of antenna gain. One of the formulas is derived based on the method of gain in the field intensity of the array antenna earlier published by Kraus, 1988, and the formula is meant for array antenna gain. The second formula can be used to determine the gain of all antennas. It is derived based on the existing gain-comparison method. The two formulas were used to compute the gain of a two rectangular folded array antenna, as an example, using the λ/2 antenna as the reference isotropic antenna. The results of the computation were compared with the experimental measurement results of the antenna gain. It is observed that the normalized root-mean-square errors of 0.52% and 0.3% were obtained when the method 1 and method 2 were respectively compared with the measured gain of the antenna. The two methods can be used to determine the gain of antennas.

5. *Conclusion*

It is evident from this study that the two formulas derived for antenna gain calculation can be adopted by the antenna designer as gain prediction formulas for the array antennas and general antennas as the case may be.

Figure 1: Experimental set up showing standard two rectangular folded array antenna employed in the measurements, and connected to the GSP 810 spectrum analyzer as the receiver.

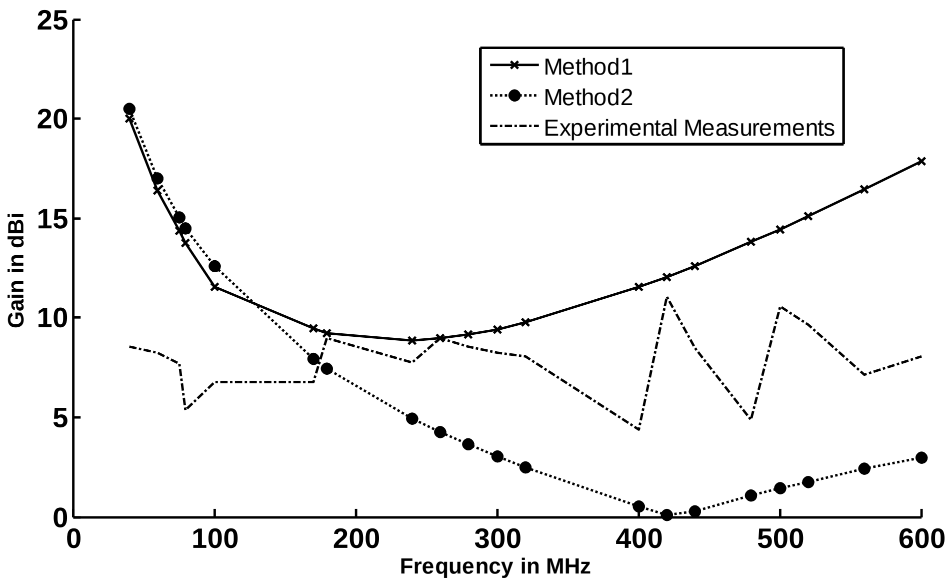

Figure 2: Showing the plot of gain versus frequency to compare the two formulas (methods 1 and 2) with the experiment.

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