Adomian Decomposition Method for the Solution of Boundary Layer Convective Heat Transfer Flow over a Flat Plate

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Abstract

This paper examines the boundary layer convective heat transfer flow over a flat plate. The nonlinear equations of momentum and energy are transformed from partial differential equations into ordinary differential equations by using similarity transformation; the results are compared with Numerical method (NM) and Homotopy perturbation method (HPM). Conclusion can be drawn that ADM provides highly precise numerical solution for nonlinear differential equations. The results were accurate especially for $\eta \leq 4$, *a general equating terms of Eckert number (EC) is derived which can be used to investigate velocity and temperature profiles in boundary layer. The effect of various flow parameters are presented and discussed as they affect the flow.*

Keywords: Adomian Decomposition Method (ADM), Convection heat transfer, Boundary layer.

1. Introduction

In many industrial applications, the problems related to forced convection in large pipes or on the surface of the turbo machine-blades can be reduced to an external boundary-layer problem over a flat plate or a wedge. The study of hydrodynamic flow and convection heat transfer has gained considerable attention due to its vast applications in industry and its important bearings on several technological and natural processes. The study of the flow and heat transfer in fluid past a porous surface has attracted the interest of many scientific investigators in view of its applications in engineering practice, particularly In chemical industries, such as the cases of boundary layer control, transpiration cooling and gaseous diffusion (Makinde et al 2007). Hayat et al. (2009) investigated the hydro magnetic oscillatory flow of a fluid bounded by a porous plate when the entire system rotates about axis normal to the plate and the result showed that the flow field is appreciably influenced by the material parameter of the Second grade fluid, applied magnetic field, the imposed frequency, rotation, suction and blowing parameters. Yurusoy and Pakdemirli (1999) examine the exact solution of boundary layer equations of a non-Newtonian fluid over a stretching sheet by the method of lie group analysis and they found that the boundary layer thickness increases when the non-Newtonian behaviour increases.

They also compared the results with that of Newtonian fluid. Ibrahim et al (2005) investigated the method of similarity reduction for Problems of radioactive and magnetic field effect on free convection and mass transfer flow past a semi-infinite flat plate. They obtained new similarity reductions and found an analytical Solution for the uniform magnetic field by using lie group method. They also presented the Numerical results for the non-uniform magnetic field. Islam et. al. (2011) applies Optimal Homotopy Asymptotic Method (OHAM) to compute an approximation to the forced convection over a horizontal flat plate. In all of the above mentioned studies, fluid viscosity was assumed to be constant. However, it is known that the physical properties of fluid may change significantly with temperature. For lubricating fluids, heat generated by internal friction and the corresponding rise in temperature affects the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. Therefore, to predict the flow behavior accurately it is necessary to take into account the viscosity variation for incompressible fluid. In this work using similarity variable and similarity solutions, a third order and a second order coupled ordinary differential equation system corresponding to the momentum and the energy equations are derived. These equations are solved using Adomian decomposition method.

The effects of the temperature- dependent fluid viscosity parameter and the influence of prandtl number on temperature fields of the fluid are investigated and analyzed with help of their graphical representations.

2. Adomian Decomposition Method

In the ADM a differential equation of the following form is considered

$Lu + Ru + Nu = g(x)$ (1)

Where L is the linear operator which is the highest order derivatives, R is the remainder of linear operator including derivatives of less order than L , Nu represents the nonlinear terms and g is the source term. Applying the inverse operator L^{-1} and rearranging gives

$u = L^{-1}g(x) - L^{-1}(Ru) - L^{-1}(Nu)$ (2)

After integrating the source term and combining it with the terms arising from given conditions , a function $f(x)$ is defined in the equation as

$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu)$ (3)

The nonlinear operator $Nu = f(u)$ is represented by an infinite series of Adomian polynomials for the specific nonlinearity. Assuming Nu is analytic we write

$$F(u) = \sum_{n=0}^{\infty} A_n \quad (4)$$

The polynomialsA_n's are generated for all kinds of nonlinearity so that they depend only on u_0 to u_k components and can be produced by the following Algorithm.

$A_0 = f(u_0)$ (5)

$A_1 = u_1 (\frac{d}{du_0})$ f (u_0) (6)

$A_2 = {}_2 u (\frac{d}{du_0})$ f $({}_0) u + \frac{u_1^2}{2} \frac{d^2}{du_0^2}$ f (u_0) (7)

⋮

The solution $u(x)$ is defined by the following series

$$u = \sum_{n=0}^{\infty} u_n \quad (8)$$

Where the components are determined recursively as follows;

$u(0) = f(x)$ (9)

$u_{n+1} = - L^{-1}(Ru_n) - L^{-1} \geq 0$ (10)

3. Governing Equations

The mathematical model for the assumed physical problem is prescribed by conservation equation of mass, momentum and energy equations. The set equations are:

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (11)

$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + v \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$ (12)

$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{V}{CP} (\frac{\partial u}{\partial y})^2$ (13)

Subject to boundary conditions:

U=0, v=0, T=T_W at y=0 (14)

U→ U_∞, T→ T_∞ as y→ ∞ (15)

In order to obtain the similarity solution of the problem, we introduce the following non-dimensional variables as defined by (Kay and Crawford 1993), Esmaeilpour and Ganji (2007).

$\eta = \frac{y}{x}\sqrt{Re_x}$ (16)

$\varphi = \sqrt{vxu_\infty}F(\eta)$ (17)

$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty}$ (18)

Where θ non-dimensional form of temperature and the Reynolds number is defined as:

$\mathrm{Re}_x = \frac{u_\infty x}{v}$ (19)

Using equations (16) we obtain,

$\eta = y\sqrt{\frac{u_\infty}{vx}}$ (20)

$a = \sqrt{\frac{u_\infty}{v}}$ (21)

$b = \sqrt{vu_\infty}$ (22)

Where a and b are constant of η and $F(\eta)$

Therefore, $\eta = a\frac{y}{\sqrt{x}}$ $\varphi = b\sqrt{x}F(\eta)$ (23)

Introducing stream function, $u = \frac{\partial\varphi}{\partial y}$ and $v = -\frac{\partial\varphi}{\partial x}$ (24)

The governing equations can be reduced to two equations where f and θ are function of the similarity variables (η):

$f''' + \frac{1}{2}ff'' = 0$ (25)

$\varepsilon\theta'' + EC(f'')^2 + \frac{f\theta'}{2} = 0$ (26)

Where $\varepsilon = \frac{1}{p_r}$, and $EC = \frac{(u_0 x)^2}{cp}\frac{1}{(T_W - T_\infty)}$ (27)

Boundary conditions:

$f(0)=0, f'(0) = 0, \theta(0) = 1, f'(\infty) = 1, \theta(\infty) = 0$ (28)

3.1 Adomian decomposition solution to the equations

$f''' = -\frac{1}{2}ff''$ (29)

$\theta'' = -\frac{ECf''^2}{\varepsilon} - \frac{1}{2\varepsilon}f\theta'$ (30)

$\frac{d3f}{d\eta 3} = f'''$, where $L_1 = \frac{d^3}{d\eta^3}$

$\frac{d^2\theta}{d\eta^2} = \theta''$, where $L_2 = \frac{d^2}{d\eta^2}$

Then we have

$F = F(0) + \eta F'(0) + \frac{\eta^2}{2}F''(0) - \frac{1}{2}L_1^{-1}ff''$ (31)

$\theta = \theta(0) + \eta\theta'(0) - L_2^{-1}\frac{ECf''^2}{\varepsilon} - L_2^{-1}\frac{1}{2\varepsilon}f\theta'$ (32)

Where $L_1^{-1} = \iiint(.)d_\eta\, d_\eta\, d_\eta$ and $L_2^{-1} = \iint(.)\, d_\eta\, d_\eta$

From the boundary conditions Equations (28) and taking $F''(0) = \alpha, \theta_0' = \beta$

And ADM solution can be obtained by:

$F(\eta) = \frac{\eta^2}{2}\alpha + L_1^{-1}\left(-\frac{1}{2}ff''\right)$ (33)

$\theta(\eta) = 1 + \eta\beta + L_2^{-1}\left(-\frac{ECf''^2}{\varepsilon}\right) + L_2^{-1}\left(-\frac{1}{2}\frac{f\theta'}{\varepsilon}\right)$ (34)

ADM is introduced in the following expression:

$F(\eta) = \sum_{m=0}^{\infty} F_m(\eta)$ (35)

$\theta(\eta) = \sum_{n=0}^{\infty}\theta_n(\eta)$ (36)

The ADM defined the nonlinear function by an infinite series of polynomials:

$P(F(\eta)) = \sum_{m=0}^{\infty} A_m$ (37)

$Q(\theta(\eta)) = \sum_{n=0}^{\infty} B_n + \sum_{n=0}^{\infty} C_n$ (38)

Adomian polynomials A_m, B_n, C_n represent the nonlinear term $P(F(\eta))$, $Q(\theta(\eta))$ and it can calculated from:

$A_m = \sum_{v=0}^{m} F_{m-v}F''_v$ (39)

$B_n = \sum_{v=0}^{n} F''_{n-v}F''_v$ (40)

$C_n = \sum_{v=0}^{n} F_{n-v}\theta'_v$ (41)

Substituting Equations (39) to (41) into Equations (33) and (34) yields

$\sum_{m=0}^{\infty} F_m(\eta) = \frac{\eta^2}{2}\alpha + L_1^{-1}\left(\sum_{m=0}^{\infty} A_m\right)$ (42)

$\sum_{n=0}^{\infty} \theta_n(\eta) = 1 + \eta\beta + L_2^{-1}\left(\sum_{n=0}^{\infty} B_n\right) + L_2^{-1}\left(\sum_{n=0}^{\infty} C_n\right)$ (43)

$F_0(\eta)$ and $\theta_0(\eta)$ are defined from the boundary condition of Equations (28) at $\eta = 0$ we have

$f_0(\eta) = \frac{\eta^2}{2}\alpha$ (44)

$\theta_0(\eta) = 1 + \eta\beta$ (45)

For determination of the other components of $F(\eta)$ and $\theta(\eta)$, we have:

$F_{m+1}(\eta) = L^{-1}(A_m)$ m=0, 1, 2, … (46)

$\theta_{n+1}(\eta) = L_2^{-1}(B_n) + L_2^{-1}(C_n)$ n= 0, 1, 2… (47)

By using Equations (39) to (41), we obtain the following terms of Adomian polynomials A_m:

$A_0 = -\frac{1}{2} f_0 f_0''$ (48)

$A_{1=} = -\frac{1}{2}\left(f_1 f_0'' + f_0 f_1''\right)$ (49)

$A_{2=} -\frac{1}{2}\left(f_2 f_0'' + f_1 f_1'' + f_0 f_2''\right)$ (50)

And this can be calculated until A_m

By ADM we obtain,

$F_0 = \frac{\eta^2}{2}\alpha$ (51)

$F_1 = -\frac{1}{240}\alpha^2\eta^5$ (52)

$F_2 = \frac{11}{161280}\alpha^3\eta^8$ (53)

$F_3 = -\frac{375}{39916800}\alpha^4\eta^{11}$ (54)

Finally

$F(\eta) = \frac{\eta^2}{2}\alpha - \frac{1}{240}\alpha^2\eta^5 + \frac{11}{161280}\alpha^3\eta^8 - \frac{5}{4257792}\alpha^4\eta^{11}$ (55)

And similar to the above we obtain,

$B_0 = -\frac{EC}{\varepsilon}\left(F_0''\right)^2$ (56)

$C_0 = -\frac{1}{2\varepsilon}\left(F_0\theta_0'\right)$ (57)

$B_1 = -\frac{EC}{\varepsilon}\left(2F_0''F_1''\right)$ (58)

$C_1 = -\frac{1}{2\varepsilon}\left(F_0\theta_1' + F_1\theta_0'\right)$ (59)

$B_2 = -\frac{EC}{\varepsilon}\left((F_1'')^2 + 2F_0''F_2''\right)$ (60)

$C_2 = -\frac{1}{2\varepsilon}\left(F_0\theta_2' + F_1\theta_1'' + F_2\theta'_0\right)$ (61)

And this can be calculated until B_n and C_n

By ADM we obtain:

$\theta_0(\eta) = 1 + \eta\beta$ (62)

$\theta_1 = -\frac{1}{48}\frac{\eta^2\alpha\left(24EC\alpha - \eta^2\beta\right)}{\epsilon}$ (63)

$\theta_2 = \frac{1}{20160}\frac{\eta^5\alpha^2(168EC\alpha\epsilon + 10\eta^2\beta + 252EC\alpha + \eta^2\beta\epsilon}{\epsilon^2}$ (64)

$\theta_3 = -\frac{1}{29030400}\frac{1}{\varepsilon^3}\left(\eta^8\alpha^3(7560\mathrm{EC}\alpha\varepsilon^2 + 280\eta^2\beta + 84\eta^2\beta\epsilon + 6480\mathrm{EC}\alpha\varepsilon + 8100\mathrm{EC}\alpha + 11\eta^2\beta\varepsilon^2)\right)$ (65)

and finally

$\theta(\eta) = -\frac{1}{29030400}\frac{1}{\varepsilon^3}\left(-29030400\varepsilon^3 - 29030400\eta\beta\varepsilon^3 + 14515200EC\alpha^2\eta^2\epsilon^2 + 604800\eta^4\alpha\beta\varepsilon^2 - 241920\mathrm{EC}\alpha^3\eta^5\epsilon^2 - 14400\alpha^2\eta^7\beta\epsilon - 362880EC\alpha^3\eta^5\epsilon - 1440\alpha^2\eta^7\beta\epsilon^2 + 7560EC\alpha^4\eta^8\epsilon^2 + 280\alpha^3\eta^{10}\beta + 84\alpha^3\eta^{10}\beta\epsilon + 6480EC\alpha^4\eta^8\epsilon + 8100EC\alpha^4\eta^8 + 11\alpha^3\eta^{10}\beta\epsilon^2\right)$ (66)

4. Results and Discussions

The results of Homotopy Perturbation Method (HPM) (Ganji et al 2009) and Numerical Method (NM) (Bejan 1995) for $F(\eta)$, $F^{'}(\eta)$ with assumed value α =0.332057 and the results of HPM (Esmaeilpour and Ganji, 2007) and NM (Bejan, 1995) for $\theta(\eta)$are compared with the ADM.

Table 4.1 The results of ADM, HPM and NM for $F(\eta)$

η	ADM	HPM	NM
0	0.000000	0.00000	0.000000
0.2	0.006641	0.00697	0.006641
0.4	0.026560	0.027876	0.026676
0.6	0.059735	0.062696	0.059722
0.8	0.106108	0.111374	0.106108
1.0	0.165572	0.173802	0.165572
1.2	0.237948	0.249804	0.237949
1.4	0.322981	0.339122	0.322982
1.6	0.420320	0.441401	0.420321
1.8	0.529517	0.55618	0.529518
2.0	0.650022	0.682883	0.650024
2.2	0.781188	0.820821	0.781193
2.4	0.922273	0.969187	0.92229
2.6	1.072457	1.127077	1.072506
2.8	1.230845	1.293501	1.230977
3.0	1.396471	1.467413	1.396808
3.2	1.568288	1.647758	1.569095
3.4	1.745122	1.83352	1.74695
3.6	1.925590	2.023791	1.929525
3.8	2.107937	2.217865	2.11603
4.0	2.289779	2.415336	2.305746

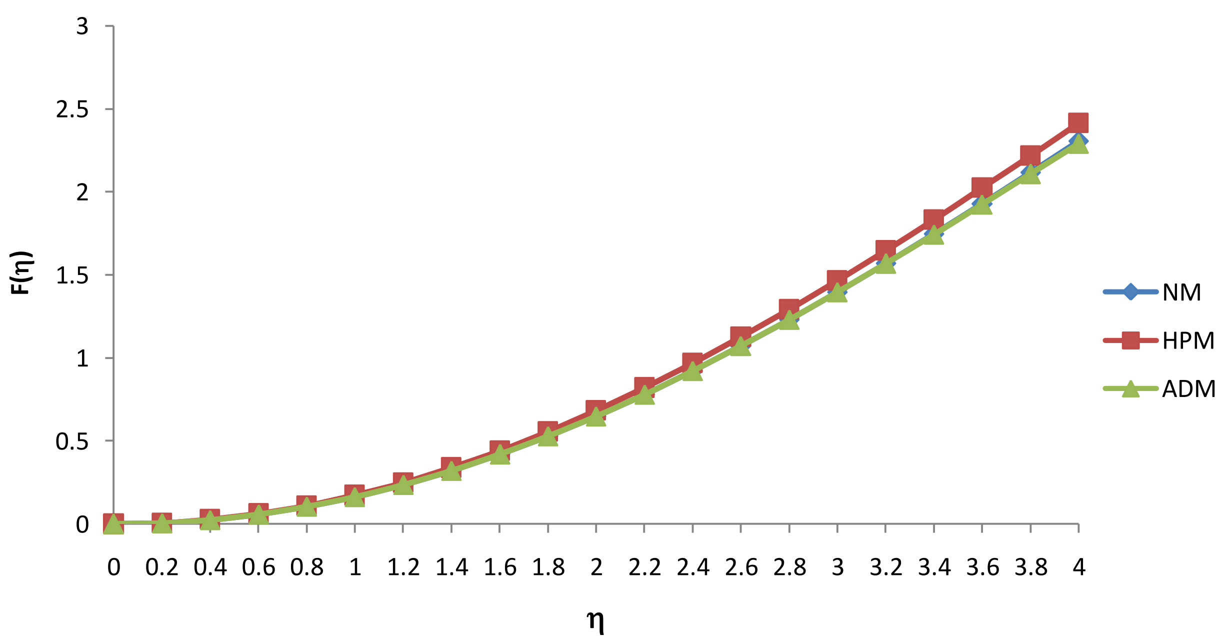

Figure 4.1: The comparison of the answer resulted by ADM, HPM NM for $\boldsymbol{F(\eta)}$

Table 4.2 The results of ADM, HPM and NM for $F'(\eta)$

η	ADM	HPM	NM
0	0	0	0
0.2	0.066408	0.066408	0.066408
0.4	0.132764	0.132764	0.132764
0.6	0.198937	0.198937	0.198937
0.8	0.264709	0.264709	0.264709
1.0	0.329780	0.329780	0.32978
1.2	0.393776	0.393776	0.393776
1.4	0.456261	0.456261	0.456262
1.6	0.516756	0.516756	0.516757
1.8	0.574755	0.574755	0.574758
2.0	0.629756	0.629756	0.629766
2.2	0.681280	0.68128	0.68131
2.4	0.728891	0.728891	0.728982
2.6	0.772204	0.772204	0.772455
2.8	0.810869	0.810869	0.81151
3.0	0.844521	0.844521	0.846044
3.2	0.872672	0.872672	0.876081
3.4	0.894529	0.894529	0.901761
3.6	0.908710	0.90871	0.92333
3.8	0.912811	0.912811	0.941118
4.0	0.902800	0.90280	0.955518

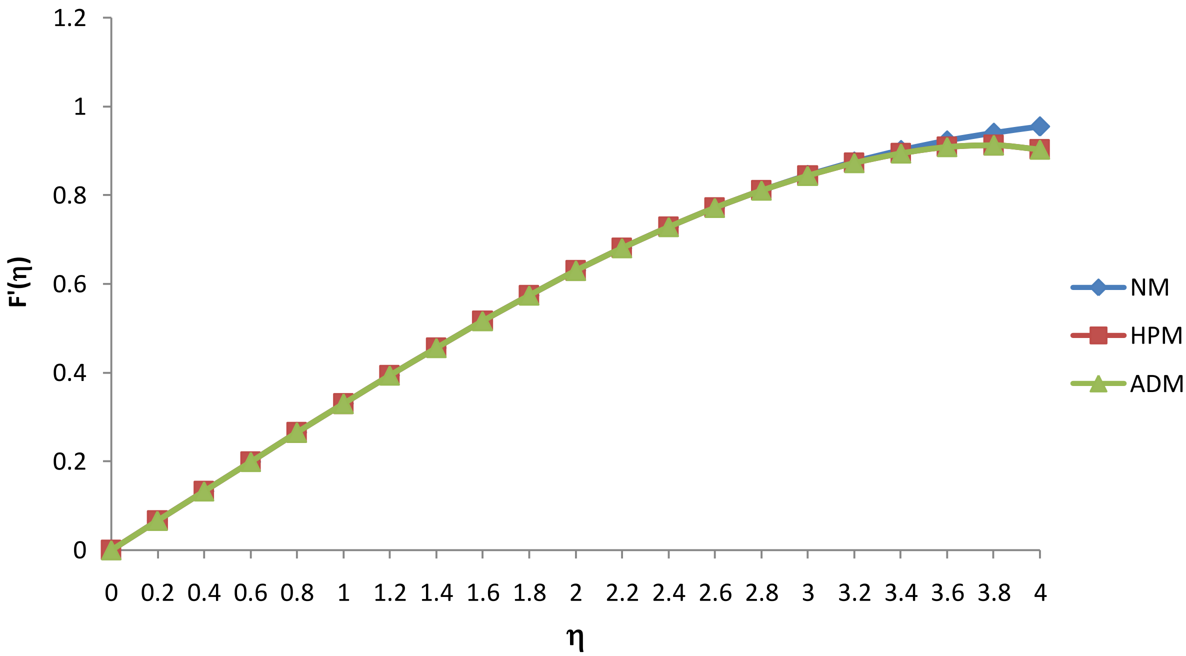

Figure 4.2 The comparison of the results by ADM, HPM and NM for $F'(\eta)$

Table 4.3. The results of ADM, HPM and NM for $\theta(\eta)$ at EC=0.0

η	ADM	NM	HPM
0	1	1	1
0.2	0.930251	0.933592	0.930302
0.4	0.860555	0.867236	0.860656
0.6	0.791052	0.801063	0.791199
0.8	0.721971	0.735291	0.72212
1.0	0.653626	0.67022	0.653746
1.2	0.586410	0.606224	0.586446
1.4	0.520780	0.543738	0.520669
1.6	0.457241	0.483243	0.456925
1.8	0.396323	0.425242	0.430655
2.0	0.338554	0.370234	0.337791
2.2	0.284438	0.31869	0.283571
2.4	0.234432	0.271018	0.233677
2.6	0.188939	0.227545	0.18862
2.8	0.148328	0.18849	0.148818
3.0	0.112982	0.143955	0.114567
3.2	0.083415	0.123918	0.095999
3.4	0.060458	0.088239	0.063049
3.6	0.045556	0.06667	0.055428
3.8	0.041260	0.058882	0.032602

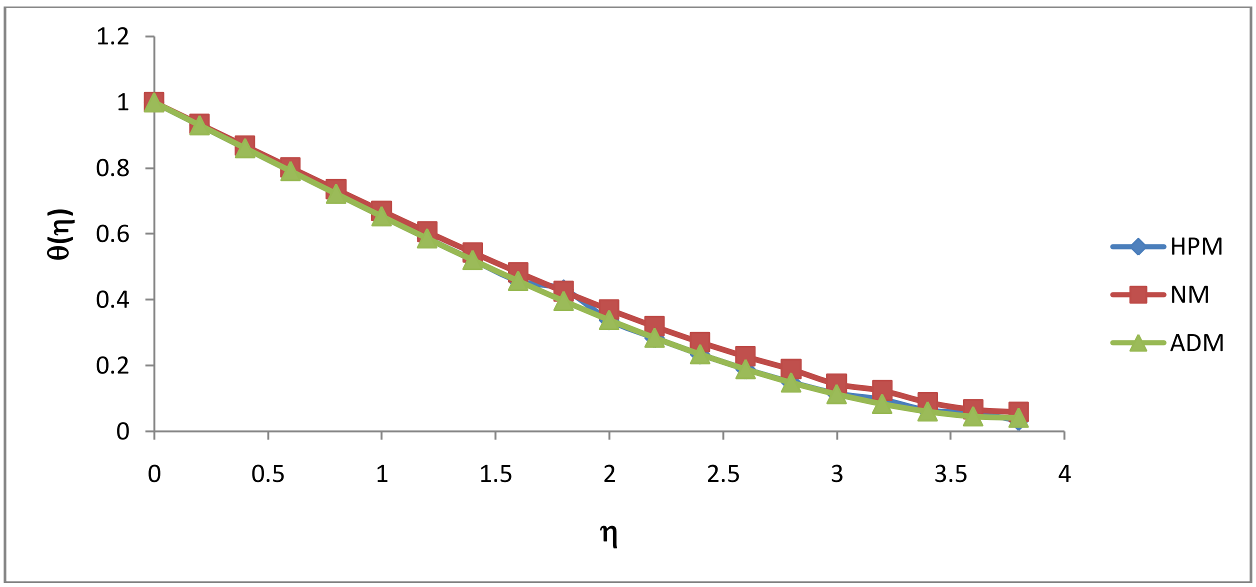

Figure4. 3 comparison of the results by ADM, HPM and NM for $\theta(\eta)$ at EC=0

Table 4.4. for $\theta(\eta)$ at *EC*=0.2 and variation of prandtl numbers

η	*Pr* =1.0	*Pr* =1.2	*Pr* =1.4
0	1	1	1
0.2	0.9298096	0.9297222	0.9296348
0.4	0.8587927	0.8584527	0.8581128
0.6	0.7870948	0.7863673	0.7856403
0.8	0.7149639	0.7137654	0.7125690
1.0	0.6427503	0.6410706	0.6393964
1.2	0.5709034	0.5688246	0.5667577
1.4	0.4999615	0.4976731	0.4954057
1.6	0.4305341	0.4283406	0.4261770
1.8	0.3632764	0.3615949	0.3599457
2.0	0.2988566	0.2982020	0.2975646
2.2	0.2379159	0.2388746	0.2398010
2.4	0.1810269	0.1842203	0.1872777
2.6	0.1286551	0.1347007	0.1404344
2.8	0.0811317	0.0906140	0.0995335
3.0	0.0386508	0.0521253	0.0647440
3.2	0.0013068	0.0193688	0.0363453

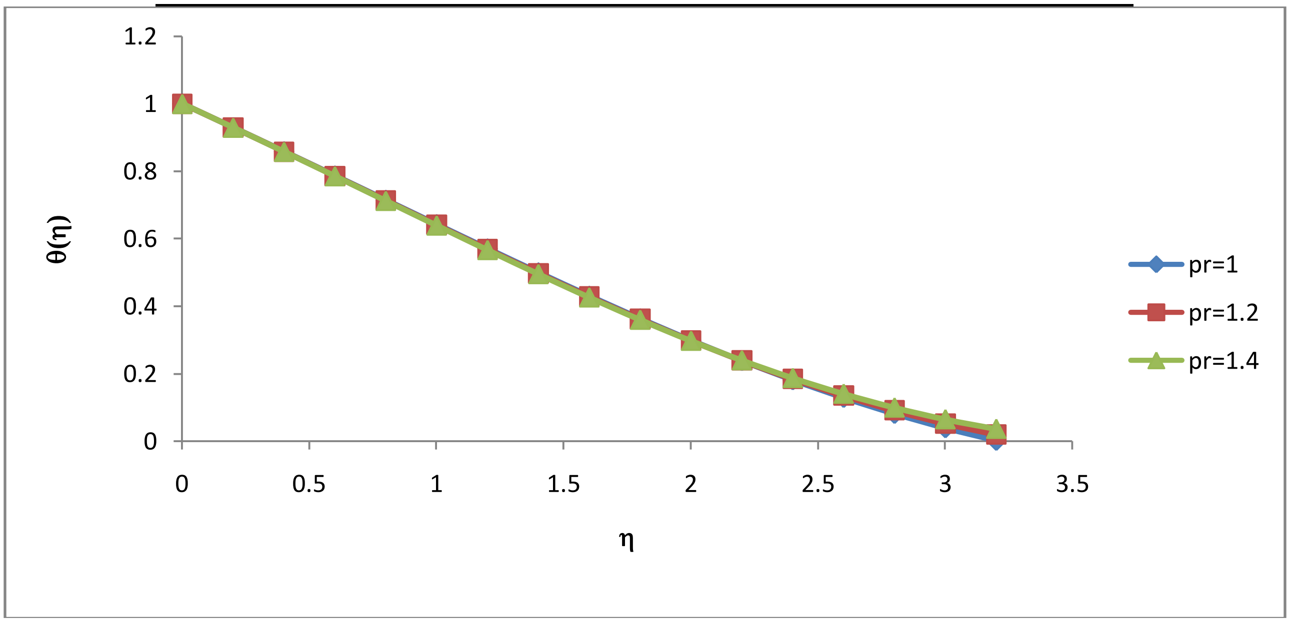

Figure 4. 4. Comparison of the results by ADM, HPM and NM $\theta(\eta)$ at $pr = 1.0, 1.2\ and\ 1.4$

4. Discussion of results

The result for f are presented in figures 4.1 and Table 4.1 also the result of f' (the velocity profile) is presented in figure 4.2 and Table 4.2, the temperature θ is evaluated for Eckert number (*EC*=0.0) in Table 4.3 and figure 4.3 the results proved that the answers for velocity and temperature distributions shows a good agreement with other method particularly with numerical solution. Effects of prandtl and Eckert number parameters on the temperature profile are shown in Table 4.4 and figure 4.4. It was observed temperature decreases with a slight increase in prandtl and Eckert number.

5. Conclusion

In this paper, Adomian Decomposition Method has been successfully applied to natural convection heat transfer problem with specified boundary conditions for momentum and energy equations. The results were compared with ones from Homotopy Perturbation Method and Numerical Method .The excellent agreement with those obtained numerically. Also, the velocity and temperature profiles were obtained as a function of η, and *pr* number. Using ADM method an attempt has been made to show the effect of *pr* number in temperature and heat transfer boundary layer. This new method accelerated the convergence to solutions. The ADM provide efficient alternative tools in solving nonlinear models.

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