Mathematics without sin α, cos α (When Angle α is Being Measured in Degrees) and π.

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Abstract

Consequently, the numeral meaning of sinα is rounded down twice – the finite number of irrational quantity π after comma is changed into the finite number of figures and a finite figure is taken from sine function endless line. In the case sin and cos are within complex mathematical expressions, the methods of analytical calculating cannot be used, because sin and cos have no analytical expressions.

Key words: h-geometry, computer-based time, space mechanics.

Introduction

Mathematics is a kind of sciences having one of the longest durations in history. The truth is, that the word mathematics has originated comparatively recently. Theretofore, various methods of calculation had been created before their integration into one branch of sciences called mathematics. Different authors define the conception of mathematics differently. We are going to use the definition of mathematics that is given in Encyclopaedia Britannica (www.britannica.com). The definition of mathematics and a review of mathematics history are presented here. **Mathematics** is the science of structure, order, and relation that has evolved from elemental practices of counting, measuring, and describing the shapes of objects.

It is supposed that astronomers suggested angle measuring using degrees several hundred years before Christ. A circle was divided into 360 parts, and the size of angle, formed by two lines or rays diverging from a common point (equal in magnitude to $^{1}/_{360}$ of a complete revolution) was called a degree. An angle of one degree leans on the length of circle hoop and amounts to $^{1}/_{360}$ part of circle length. The expression of a link between the diameter of a circle and the length of a circle has been sought for a long time. After long search it was established that a proportion between the length of a circle and its diameter is equal to irrational quantity π. Quantity π is mostly used in converting the size of angle in degrees to the size of angle in radians. As it can be seen, the origin of irrational quantity π is related to measurements of the length of a circle and its diameter. In geometrical meaning, the size of an angle expressed in radians means that the size of an angle is being measured in the length of a circle hoop.

The question appears – is there any other method to express the size of an angle?

The other none the less popular conception in mathematics is a sine. A sine is the ratio of the length of the side opposite an acute angle to the length of the hypotenuse (radius of a circle) in a triangle. The angle that is in front of the side of a triangle is marked α. In such case the sine of an angle is written as sinα. It appeared that sinα had no analytical expression, consequently, the expressions of sinα, while angle α was changing, were presented in mathematical numerical tables. Only later it has been discovered how to spread out the function sinα in an endless line.

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \ldots\ldots\ldots\ldots$$

Functions cos and tan are spread out likewise.

Therefore, nowadays sinα is calculated spreading out the function in an endless line. Here angle α has to be translated from degrees into radians. The following formula is used:

$$\alpha = \frac{\pi}{180}\cdot\alpha^{o}$$

where π is irrational quantity. Its numeral meaning

$$\pi = 3.141592653589793............$$

has been calculated for a long time and nowadays a new record has been achieved – 5 trillion figures after comma.

Is it the unique method to form mathematical methods of systems if a system has angles?

As far back as 1987 the present writer offered to measure the size of an angle using not the length of a circle hoop but a side (straight). The height of such a side varies from 0 to1. The height of a side is indicated as letter h. The height h has analytical expression depending on the size of an angle. However, in such case a sine that is indicated as sph has algebraic analytical expression.

After coming to the angle measurements applying h parameters, one can forget about measurements applying degrees (radians). Angles are going to be measured applying h parameter gauge. If an angle changes from 0 to 90 degrees, the parameter h changes from 0 to 1.

Angle calculation

Classics

Let us take a rectangular triangle, whose perpendicular lines are a and b. As in astronomy, in navigation the size of angle α of such triangle is calculated in radians, using the classical formula of trigonometry

$$\alpha = \operatorname{atan}\left(\frac{a}{b}\right) \tag{1}$$

atan function has no analytical expression and can be calculated by outspreading it in infinite line

$$\operatorname{atan}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$$

Definition of the size of an angle, used in h – geometry**.**

The size of an angle in h –geometry [1] is measured in h – parameters, where

$$0 < h < 1$$

We will determine the angle in front of perpendicular line of the same rectangular triangle with the expression

$$h = \frac{a}{a+b} \tag{2}$$

The link between the h parameter and the size of an angle measured in radians α is determined by the formula

$$\alpha = \operatorname{atan}\left(\frac{h}{1-h}\right)$$

or

$$h = \frac{\tan(\alpha)}{1+\tan(\alpha)}$$

h-geometry

It is the basis of h-geometry [1] www.rdi.lt. The principles of h-geometry were discussed at the Institute of Mathematics and Informatics of Vilnius University of October 2009.

Trigonometry functions in h-geometry are indicated as sph, cph and tph. They have algebraic meanings.

$$sph = \frac{h}{\sqrt{h^2 + (1-h)^2}} = z \qquad h = \frac{z}{z + \sqrt{1-z^2}} \tag{3}$$

$$cph = \frac{1-h}{\sqrt{h^2 + (1-h)^2}} = z \qquad h = \frac{\sqrt{1-z^2}}{z^2 + \sqrt{1-z^2}} \tag{4}$$

Also

$$tph = \frac{h}{1-h} = z \qquad h = \frac{z}{1+z} \tag{5}$$

As it can be seen every trigonometric functions in h-geometry have algebraic meanings. The relation between an angle of classical trigonometric function measured by radians and h-parameters is established

$$h = \frac{\tan(\alpha)}{1+\tan(\alpha)}$$

What is the meaning of that to practical calculation?

Example

We will calculate using Mathcad15 program. Given

$a = 2 \qquad b = 5$

From (1) we will get

$\alpha = 0.38050637$

Under the same values of a and b (2) we will calculate the h – parameter

$h = 0.28571428$

Then the values of trigonometrically functions (3), (4), (5) will be

$sph = 0.37139067$

$cph = 0.92847669$

$tph = 0.4$

Having calculated the trigonometrical functions with the help of Mathcad 15, when α is given (4) we will get

$\sin(\alpha) = 0.37139067$

$\cos(\alpha) = 0.92847669$

$\tan(\alpha) = 0.4$

As we see, the calculation results are fully coincident.

Several examples

The first. A panel made of reinforced concrete is being raised by a crane. Three cables are attached to the panel. Applying the methods of theoretical mechanics the equations have been written; the solution gives the answer to the question what forces affect cables in the process of raising a panel. The cable A is being affected by the force written in the following formula

$$T_A = \frac{P\cdot\sin(\beta)}{2\cdot(\cos(\alpha)\cdot\sin(\beta) + \sin(\alpha)\cdot\sin(\gamma)\cdot\cos(\beta))} \tag{6}$$

sizes of angles α, β, γ are established

$$\alpha = \operatorname{atan}\left(\frac{l}{d}\right) \quad \beta = \operatorname{atan}\left(\frac{b}{d}\right) \quad \gamma = \operatorname{atan}\left(\frac{a}{l}\right) \tag{7}$$

where

$$l = \sqrt{a^2 + b^2}$$

Angles (7) are being calculated spreading the functions atan out in endless lines. Here a, b and d are dimensions of a panel. P is weight of a panel. The meanings of trigonometric functions sin and cos in a formula (6) are also being calculated spreading the functions out in endless lines.

Whereas applying the methods of h-geometry we can write

$$T_A = \frac{P}{4}\cdot\sqrt{1 + \frac{a^2+b^2}{d^2}} \tag{8}$$

instead of (6) and (8).

The results of calculation will be identical. However, in point of computational technologies (computer-based time), calculations applying (8) are disparately simpler than calculations (6) and (7) (plus atan, con calculations).

Example. The following data are given:

$a = 1$ $b = 0.75$ $d = 2$ $P = 4$

Calculating applying formulae of classical geometry (6), (7) and h-geometry formula (8), we get an identical result

$$T_A = 1.179$$

However, in point of computational technologies and computer-based time, calculations applying mathematical model of h-geometry (8) are more effective than the other ones. Computation time measurement has shown that calculating applying (8) in comparison with (6) takes off mechanical time 5 times less.

The second Transformation of vectors' coordinates

Classics

The classical trigonometric functions sin and cos are used for the determination of vector projection on the coordinate axes, both in classical celestial mechanics and classical vector algebra. The classical trigonometric functions sin and cos, as we know, have no analytical expressions, and can only be calculated using the methodology proposed by Euler - to outspread sin and cos in the infinite line.

Often, the mathematical model of the system can be written as

$$\begin{aligned} a11\cdot x1 + a12\cdot x2 + a13\cdot x3 &= A14 \\ a21\cdot x1 + a22\cdot x2 + a23\cdot x3 &= A24 \\ a31\cdot x1 + a32\cdot x2 + a33\cdot x3 &= A34 \end{aligned}$$

Here the coefficients are expressed as

$$\begin{aligned} a11 &:= \cos(\omega)\cdot\cos(\Omega) - \sin(\omega)\cdot\cos(i)\cdot\sin(\Omega) \\ a12 &:= \cos(\omega)\cdot\sin(\Omega) + \sin(\omega)\cdot\cos(i)\cdot\cos(\Omega) \\ a13 &:= \sin(\omega)\cdot\sin(i) \\ a21 &:= -\sin(\omega)\cdot\cos(\Omega) - \cos(\omega)\cdot\cos(i)\cdot\sin(\Omega) \\ a22 &:= \sin(\omega)\cdot\sin(\Omega) + \cos(\omega)\cdot\cos(i)\cdot\cos(\Omega) \\ a23 &:= \cos(\omega)\cdot\sin(i) \\ a31 &:= \sin(i)\cdot\sin(\Omega) \\ a32 &:= -\sin(i)\cdot\cos(\Omega) \\ a33 &:= \cos(i) \end{aligned}$$

The computer usually calculates the systems of equations by an inverse matrix method.

We will search for only one solution. For example x3. It will be comprised out of three parts

$$x3 = x31 + x32 + x33$$

where

$$x31 = \frac{a21\cdot a32 - a22\cdot a31}{a11\cdot a22\cdot a33 - a11\cdot a23\cdot a32 - a21\cdot a12\cdot a33 + a21\cdot a13\cdot a32 + a31\cdot a12\cdot a23 - a31\cdot a13\cdot a22}\cdot A14$$

$$x32 = \frac{-a11\cdot a32 + a12\cdot a31}{a11\cdot a22\cdot a33 - a11\cdot a23\cdot a32 - a21\cdot a12\cdot a33 + a21\cdot a13\cdot a32 + a31\cdot a12\cdot a23 - a31\cdot a13\cdot a22}\cdot A24$$

$$x33 = \frac{a11\cdot a22 - a12\cdot a21}{a11\cdot a22\cdot a33 - a11\cdot a23\cdot a32 - a21\cdot a12\cdot a33 + a21\cdot a13\cdot a32 + a31\cdot a12\cdot a23 - a31\cdot a13\cdot a22}\cdot A34$$

Having input the aij values we get lengthy expressions, which have the functions sin and cos in them.

As we see, the calculations of x3 are difficult. x1 ir x2 are calculated likewise.

In practics usually the mathematical method is used, when transformation is performed twice. In this case $\omega = 0$.

$$\begin{pmatrix} A14 \\ A24 \\ A34 \end{pmatrix} = \begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix}\cdot\begin{pmatrix} x1 \\ y1 \\ z1 \end{pmatrix}$$

where

$$a11 = \cos(\Omega) \qquad a12 = \sin(\Omega) \qquad a13 = 0$$
$$a21 = -\cos(i)\cdot\sin(\Omega) \qquad a22 = \cos(i)\cdot\sin(\Omega) \qquad a23 = \sin(i)$$
$$a31 = \sin(i)\cdot\sin(\Omega) \qquad a32 = -\sin(i)\cdot\cos(\Omega) \qquad a33 = \cos(i)$$

x1, y1, z1 needs to be found , the values of x ,y , z are known
For example, let us find z1. Let us note that

$$z1 = z11 + z12 \qquad (9)$$

where

$$z11 = A24\cdot\frac{\cos(\Omega)^2\cdot\sin(i) + \sin(\Omega)^2\cdot\sin(i)}{\cos(\Omega)^2\cdot\cos(i)^2 + \sin(i)^2\cdot\cos(\Omega)^2 + \cos(i)^2\cdot\sin(\Omega)^2 + \sin(i)^2\cdot\sin(\Omega)^2} \qquad (10)$$

$$z12 = A34\cdot\frac{\cos(\Omega)^2\cdot\cos(i) + \sin(\Omega)^2\cdot\cos(i)}{\cos(\Omega)^2\cdot\cos(i)^2 + \sin(i)^2\cdot\cos(\Omega)^2 + \cos(i)^2\cdot\sin(\Omega)^2 + \sin(i)^2\cdot\sin(\Omega)^2} \qquad (11)$$

As it can be seen, calculations, when turned only twice, are little less complicated. But still sin and cos needs to be calculated using the infinite line.

h-geometry model
We will get h-geometry model if we refuse the angle measurement in degrees, then instead of sinα we can write sph, and instead of cosα we can write cph, where h - is the size of the angle, measured by the h-parameters.

$$\begin{pmatrix} cph2 & sph2 & 0 \\ -cph1\cdot sph2 & cph1\cdot cph2 & sph1 \\ sph1\cdot sph2 & -sph1\cdot cph2 & cph1 \end{pmatrix}$$

Sizes of angles h can be measured directly. For equalization of h-geometry model and the classical model (α), the parameters h may be converted from α parameters using the link formulas

$$h1 = \frac{\tan(i)}{1 + \tan(i)}$$

$$h2 = \frac{\tan(\Omega)}{1 + \tan(\Omega)}$$

Having input the values of matrix coefficients, which are expressed as sph and cph in (1),(2) we will get

$$z1 = \frac{A24\cdot h1 + A34\cdot(1 - h1)}{\sqrt{h1^2 + (1 - h1)^2}} \qquad (12)$$

As we can see, instead of (9),(10),(11),(12) formulas, we will have to use only one algebraically expression (12).
Let us take the same example, just using the expression (12)
Example

Let us take a specific example. Calculated using Mathcad 15. Given

$$A14 = 7\ ,\ A24 = 5\ ,\ A34 = 6$$

$$\Omega = 0{,}035 \qquad i = 0{,}052$$

From (9),(10),(11),(12) we will get

$$z1 = 6{,}251773 \qquad (13)$$

as we can see, we have to make sufficiently many calculations, in order to find the value of z1
For equalization we will use the same i value, but just calculate it into h size. We will get

$$h1= 0{,}049472052$$

By using formula (12) we will get the value of z

$$z1= 6{,}251773 \qquad (14)$$

Having compared the calculations using the functions of classical geometry sin and cos, with the result , using the model (12), it shows that the values of digital calculation completely coincide.

However, using the h-geometry functions sph and cph the calculation procedures (calculation period) (12) are much shorter.

Conclusion

Withdraw functions of classical geometry sinα and cosα, and transit to h-geometry functions sph and cph and tph that grund save computing time 3–5 times.
We offer to use h-geometry functions sph and cph, which have analytical expressions, instead of now used classical functions sinα and cosα, therefore the mathematic models of matrix take analytical expressions, and the calculations procedures are significantly simplified. Other examples of calculation, see [1]

Literature:

1. Donaldas Zanevicius. h- GEOMETRY. Neo- sines in space mechanics.
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