Maximum Pivot Angle for a Small Element of a Concave Spherical Mirror Illuminated from Its Center of Curvature

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Abstract

A concave spherical mirror, composed of a large number of infinitesimally small reflective elements and illuminated by a source located at its center of curvature, is considered. The maximum angle through which an arbitrary element may pivot about a meridian before its reflected ray grazes the periphery of the mirror's aperture is derived geometrically; this angle is found to be independent of the mirror's radius of curvature. Corrections are derived for the maximum pivot angle and maximum angular displacement from the principal axis for small but finite-size elements. These results may be applicable to propulsive mirrors (solar sails) in space.

Keywords: applied geometrical optics – concave mirror geometry – solar sails

1. Introduction

Consider a concave spherical mirror, idealized as an infinitesimally thin spherical cap with radius of curvature R and aperture angle Ψ, illuminated by a point-source of monochromatic light located at its center of curvature. Suppose that this mirror is not monolithic but is composed of a large number of reflective elements, each in the form of a regular polygon and so small that it may be treated as an optically flat circular disk of arbitrarily small diameter.

Next, suppose that a randomly selected element is bisected by a meridian, and that the element may pivot about this meridian. Then, the question addressed in this paper is as follows: As a function of its angular distance θ from the principal axis (the polar axis of the mirror), what is the maximum angle through which an arbitrary element may pivot (in either direction) before its reflected ray grazes the periphery (rim) of the mirror's aperture?

2. Pivot angle derivation

Figure 1 shows the mirror aperture of radius a, along with the light source S and the mirror element E; in the interests of clarity, the body of the mirror is not shown. The incident ray crosses the aperture plane at a radial distance r from the source and at an elevation angle θ relative to the principal axis. Depending on the angle through which the element has pivoted, the reflected ray will exit the aperture somewhere along the locus of points $P – P'$ comprising a chord of half-length z. On **Figure 1**, note that the points P and P' are where a reflected ray grazes the mirror's rim; since for a planar reflector the angles of incidence and reflection are equal, this defines an element's maximum pivot angle α_{max} as half the angle whose tangent is $z/(R – r)$ so that

$$\tan 2\alpha_{max} = z/(R - r). \qquad (1)$$

Then, from Pythagoras' theorem,

$$z^2 = a^2 - y^2 \qquad (2)$$

and, from **Figure 1**, we read off the relations $y = x \tan \theta$, $x = R \cos \Psi$ and $a = R \sin \Psi$. Using these to eliminate a and y from (2) gives

$$z^2 = R^2[1 - (\cos^2 \Psi/\cos^2 \theta)] \qquad (3)$$

where we have used the identity $\sec^2 \theta \equiv 1 + \tan^2 \theta$. Also, from **Figure 1** we see that

$r \cos \theta = R \cos \Psi$ (4)

Upon using (3) and (4) to eliminate z and r, respectively, from relation (1), straightforward algebra yields the result

$\alpha_{max} = ½ \arctan [(\cos \theta + \cos \Psi)/(\cos \theta - \cos \Psi)]^{½}$ (5)

While one would intuitively expect an element's maximum pivot angle to depend on the element's location on the mirror and on the mirror's aperture angle, the above result shows that α_{max} does not depend on the mirror's chief optical parameter—its radius of curvature.

3. Corrections for finite element size

So far, we have idealized the mirror elements as infinitesimally small, reflecting infinitesimally narrow beams of incident light. To extend our analysis to elements of finite (but still very small) size, we define a "smallness parameter" ε as the ratio of an element's radius, b, to that of the aperture:

$\varepsilon \equiv b/a$ (6)

(For polygonal elements, a useful approximation for b could be obtained by averaging the radii of their inscribed and escribed circles.) Since $a = R \sin \Psi$, we now have the useful relation

$b = \varepsilon R \sin \Psi$ (7)

It is clear that an element's finite size limits both its maximum pivot angle and its maximum angular distance from the principal axis. Regarding the second issue, it is obvious that an element can only be displaced from the principal axis until its edge reaches the rim of the mirror; the element's maximum angular displacement is thus reduced by its own angular half-width $\Delta\theta \approx b/R$. From (7), the limits on an element's location on its meridian are therefore

$0 \leq \theta \leq (\Psi - \varepsilon \sin \Psi)$ (8)

To address the limitation on the maximum pivot angle, we begin with a small element of radius b reflecting a beam of finite width. Assuming the beam remains sufficiently collimated when it reaches the mirror's rim, its half-width will remain b so that α_{max} will be reduced by the amount $\Delta\alpha \approx b/l$ (where l is the distance from the element to the mirror's rim). From **Figure 1**, we see that $l^2 = (R - r)^2 + z^2$. Using relations (3), (4) and (7), this gives the final result as

$\Delta\alpha = (\varepsilon/\sqrt{2})(\sin \Psi)[(\cos \theta)/(\cos \theta - \cos \Psi)]^{½}$ (9)

where, of course, θ is subject to the restriction (8).

4. Conclusion

We conclude by noting that, while no practical application of the above results to imaging mirrors is immediately obvious, there exists a class of mirrors where the redirection of incident light by pivoting elements could prove both practical and important: propulsive mirrors, *i.e.*, solar sails. We are currently researching the use of small built-in pivoting reflective elements for maneuvering the ultimate version of such a device—the Shkadov Thruster [1]. In this scheme, a megastructure spherical concave mirror (*i.e.*, a gigantic solar sail) would use the Sun as a point-source at the center of curvature, reflecting the Sun's light back onto it while sunlight diametrically opposite the mirror escapes freely. The mirror-Sun system would then constitute a Class A stellar engine, capable of moving the entire solar system through space.

References

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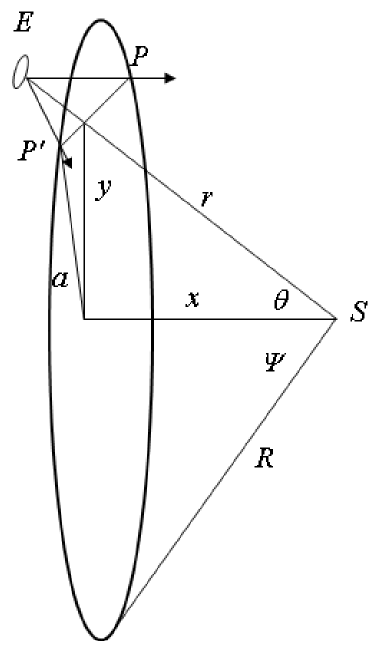

Figure 1: The mirror aperture, an arbitrary reflective element, and their associated geometric variables. (Mirror body omitted for clarity.)

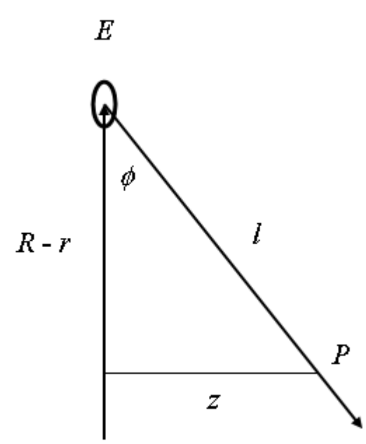

Figure 2: Incident and reflected rays at maximum pivot angle (mirror and aperture omitted); the labeled angle is $2\alpha_{max}$.