Volatility Modelling of the Nairobi Securities Exchange Weekly Returns Using the Arch-Type Models

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Abstract

In this paper we identify the most efficient ARCH-type model that can be applied to the Nairobi stock exchange data for forecasting and prediction of volatility which in turn is important in pricing financial derivatives, selecting portfolios, measuring and managing risks more accurately. The establishment of an efficient stock market is indispensable for an economy that is keen on utilizing scarce capital resources to achieve its economic growth. The purpose of this study was to determine the most efficient model from the symmetric and the asymmetric GARCH models. The models were evaluated by use of the Shwartz Bayesian Criteria (SBC), Akaike Information Criteria (AIC) and the Mean Squared Error (MSE). The results show that the AR-Integrated GARCH (IGARCH) models with student's t-distribution are the best models for modelling volatility in the Nairobi Stock Market data.

Key words: ARCH, Model Efficiency, MSE, Volatility

1.0 Introduction

Stock market volatility is one of the most important aspects of financial market developments, providing an important input for portfolio management, option pricing and market regulation (Poon and Granger, 2003). An investor's choice of a portfolio is intended to maximize the expected return subject to a risk constraint, or to minimize his risk subject to a return constraint. An efficient model for forecasting of an asset's price volatility provides a starting point for the assessment of investment risk. To price an option, one needs to know the volatility of the underlying asset. This can only be achieved through modelling the volatility. Volatility also has a great effect on the macro-economy. High volatility beyond a certain threshold will increase the risk of investor loses and raise concerns about the stability of the market and the wider economy (Hongyu and Zhichao, 2006).

Financial time series modelling has been a subject of considerable research both in theoretical and empirical statistics and econometrics. Numerous parametric specifications of ARCH models have been considered for the description of the characteristics of financial markets. Engle (1982) introduced the Autoregressive Conditional Heteroscedasticity (ARCH) for modelling financial time series while Bollerslev (1986) came up with the Generalized ARCH (GARCH) to parsimoniously represent the higher order ARCH model while Nelson (1991) introduced the Exponential GARCH to capture the asymmetric effect.

Other specifications of the GARCH model includes: the TGARCH introduced by Zakoian (1994), IGARCH by Engle and Bollerslev (1986), the Quadratic GARCH (QGARCH) model introduced by Sentana (1995), the GJR model by Glosten *et al.*, (1993) just to mention but a few.

The Sub-Saharan Africa has been under-researched as far as volatility modelling is concerned. Studies carried out in the African stock markets include, Frimpong and Oteng-Abayie (2006) who applied GARCH models to the Ghana Stock Exchange, Brooks *et al.*, (1997) examined the effect of political change in the South African Stock market, Appiah-Kusi and Pascetto (1998) investigated the volatility and volatility spillovers in the emerging markets in Africa. More recently, Ogum *et al.,* (2006) applied the EGARCH model to the Kenyan and Nigerian Stock Market returns. From the available literature, the NSE just like other Sub Saharan Africa Equity Markets has been under-researched as far as market volatility is concerned and therefore this study contributes to the small literature available on the Nairobi stock market.

There is a significant amount of research on volatility of stock markets of developed countries. The focus of financial time series modelling has been on the ARCH model and its various extensions thereby ignoring the aspect of efficiency within the ARCH-type of models. As a matter of fact, the subject of the efficiency of the models for financial modelling has received little attention as far as econometric modelling is concerned. This study therefore aims at finding the most efficient model from amongst the autoregressive conditional heteroscedasticity class of models. The remainder of this paper is arranged as follows; section 2 presents the properties of financial data, section 3 gives an overview of the ARCH-type models considered in this paper, in section 4 we present the results and discussions, section 5 highlights the summary and conclusions while section 6 contains the references.

2.0 Properties of Financial Data

Financial time series data often exhibit some common characteristics. Fan and Yao (2003) summarizes the most important features of financial time series as: The series tend to have leptokurtic distribution, i.e they have heavy tailed distribution with high probability of extreme values. In addition, changes in stock prices tend to be negatively correlated with changes in volatility, that is; volatility is higher after negative shocks than after positive shocks of the same magnitude. This is referred to as the leverage effect. The sample autocorrelations of the data are small whereas the sample autocorrelations of the absolute and squared values are significantly different from zero even for large lags. This behaviour suggests some kind of long range dependence in the data. The distribution of log returns over large periods of time (such as a month, a half a year, a year) is closer to a normal distribution than for hourly or daily log-returns. Finally, the variances change over time and large (small) changes of either sign tend to be followed by large (small) changes of either sign (Mandelbrot, 1963). This characteristic is known as volatility clustering. These are facts characterizing many economic and financial variables.

Researchers have applied different models to the stocks data from time to time. Mandelbrot (1963) utilized the infinite variance distributions when considering the models for stock market price changes. Fama (1965) similarly pointed out initially, their application in cases of economics particularly in modelling stock market prices. Fama *et al.,* (1969) used a random walk to model the price changes. Andrew and Whitney (1986) tested the random walk hypothesis for weekly stock market returns by comparing the variance estimators. Here the random walk model was strongly rejected. In recent studies, various specifications of ARCH models have been considered for the description of the characteristics of financial markets. Some studies in which ARCH-type models were utilized include; Gary and Mingyuon (2004) who applied the GARCH model to Shanghai Stock Exchange, Bertram (2004) modelled Australian Stock Exchange using ARCH models and Baudouhat (2004) used the GARCH model in analyzing the Nordic financial market integration. In addition, Curto (2002) employed the GARCH model to explain the volatility of the Portuguese equity market, Walter (2005) applied the structural GARCH model to portfolio risk management while Frimpong and Oteng-Abayie (2006) modelled the Ghana Stock Exchange volatility using the GARCH models. More over, Ogum *et al*., (2006) applied EGARCH model to the Kenyan and Nigeria daily stock market data.

3.0 Autoregressive Conditional Heteroscedasticity (ARCH) models

An ARCH process is a mechanism that includes past variances in the explanation of future variances (Engle, 2004). Autoregressive describes a feedback mechanism that incorporates past observations into the present. ARCH models specifically take the dependence of the conditional second moments in modelling consideration.

This accommodates the increasingly important demand to explain and to model risk and uncertainty in financial time series (Degiannakis and Xekalaki, 2004; Engle, 2004; Fan and Yao, 2003).

An ARCH process can be defined in terms of the distribution of the errors of a dynamic linear regression model. The dependent variable y_t is assumed to be generated by

$$y_t = x_t'\xi + \varepsilon_t \quad t = 1,\ldots,T \qquad 1$$

where x_t' is a kx1 vector of exogenous variables, which may include lagged values of the dependent variable and ξ is a kx1 vector of regression parameters. The ARCH model characterizes the distribution of the stochastic error ε_t conditional on the realized values of the set of variables $\psi_{t-1} = \{y_{t-1}, x_{t-1}, y_{t-2}, x_{t-2}, ...\}$. In practice, it is assumed that

$\varepsilon_t / \psi_{t-1} \sim N(0, h_t)$ where $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2$ with $\alpha_0 > 0$ and $\alpha_i \geq 0, i = 1,..., q$ to ensure that the conditional variance is positive (Engle's (1982). An explicit generating equation for an ARCH process is $\varepsilon_t = \eta_t\sqrt{h_t}$ where η_t ~*i.i.d* N (0,1). Since h_t is a function of ψ_{t-1} and is therefore fixed when conditioning on ψ_{t-1}, it is clear that ε_t as will be conditionally normal with $E(\varepsilon_t / \psi_{t-1}) = \sqrt{h_t}E(\eta_t / \psi_{t-1}) = 0$ and $Var(\varepsilon_t / \psi_{t-1}) = h_t$, $Var(\eta_t / \psi_{t-1}) = h_t$.

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model developed by Bollerslev (1986) is a generalized ARCH (GARCH) where the conditional variance is $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \ldots + \alpha_q\varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \ldots + \beta_p h_{t-p}$ with the inequality conditions $\alpha_0 > 0$, $\alpha_i \geq 0$ for *i=1,…,q*, $\beta_i \geq 0$ for *i= 1,…,p* to ensure that the conditional variance is strictly positive.

When the parameter estimates in GARCH (p,q) models are close to the unit root but not less than unit, i.e $\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j = 1$, for the GARCH process, the multi-step forecasts of the conditional variance do not approach the unconditional variance. These processes exhibit the persistence in variance/volatility whereby the current information remains important in forecasting the conditional variance. Engle and Bollerslev (1986) refer to these processes as the Integrated GARCH or IGARCH and they do not possess a finite variance but are stationary in the strong sense (Nelson, 1990). The simplest GARCH(1,1) is often found to be the benchmark of financial time series modelling because such simplicity does not significantly affect the preciseness of the outcome.

Another extension is the GARCH-M model developed by Engle *et al.,* (1987) whose key postulate was that time varying premia on different term instruments can be modelled as risk premia where the risk is due to unanticipated interest rates and is measured by the conditional variance of the one period holding yield. The GARCH (1,1)-M model is presented as $x_t = y_{t-1}\beta + h_t\gamma + \varepsilon_t$ where x_t and h_t are defined as before while y_{t-1} is a vector of additional explanatory variables. Just like the GARCH model, the GARCH-M is unable to capture asymmetric characteristics of financial data. The Exponential GARCH (EGARCH) models were introduced by Nelson (1991) in an attempt to address the two major limitations of the GARCH models. Here the volatility depends not only on the magnitude of the shock but also on their corresponding signs. The non-negativity restrictions are not imposed as in the case of GARCH since the EGARCH model describes the logarithm of the conditional variance which will always be positive. The specification for the conditional variance (Nelson, 1991) is given as,

$$Log\sigma_t^2 = \varpi_0 + \sum_{i=1}^{q}\beta_i \log\sigma_{t-i}^2 + \sum_{i=1}^{p}\alpha_i \frac{|\varepsilon_{t-i}|}{|\sigma_{t-i}|} + \sum_{i=1}^{p}\gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \qquad 10$$

Note that $\varepsilon_t = \sqrt{\eta_t\sigma_t}$ where η_t ~ i.i.d (0,1).

The parameter (α_i) in equation (10) measures the impact of innovation on volatility at time t while parameter (β_i) is the auto-regressive term on lagged conditional volatility, reflecting the weight given to previous period's conditional volatility *t*. It measures the persistence of shocks to the conditional variance.

The stationarity requirement is that the roots of the auto-regressive polynomial lie outside the unit circle. For EGARCH (1,1) this translates into $\beta_1 < 1$ (Ogum *et al*., 2006). Unlike the linear GARCH, in the EGARCH model a negative shock can have a different impact compared to a positive shock if the asymmetry parameter γ_i is non-zero.

Threshold GARCH models (TGARCH) were introduced by Zakoian (1994). The generalized specification of the conditional variance equation is given by,

$$h_t = \alpha_0 + \sum_{j=1}^{q} \beta_j h_{t-j} + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{k=1}^{r} \gamma_k \varepsilon_{t-k}^2 I_{t-k}^- \qquad 12$$

where $\varepsilon_t = \sqrt{\eta_t h_t}$ and $I_t^- = 1$, if $\varepsilon_t < 0$ and zero otherwise. In this model, good news, $\varepsilon_{t-i} > 0$, and bad news $\varepsilon_{t-i} < 0$, have differential effects on the conditional variance; good news has an impact of α_i, while bad news has an impact of $\alpha_i + \gamma_i$. If $\gamma_i > 0$, bad news increases volatility while if $\gamma_i \neq 0$, the news impact is asymmetric. When the threshold term is set to zero, then equation (12) becomes a GARCH (p,q) model.

4.0 Results and Discussions

In this study, four sets of data consisting of the weekly average share prices for Bamburi Cement Ltd, National Bank of Kenya Limited (NBK), Kenya Airways (KQ) Ltd as well as the weekly average NSE 20 share index were used. The data was obtained from the Nairobi Stock Exchange (NSE) for the period between 3rd June 1996 to 30th October 2011 for the company share prices while for the NSE 20-share index data was for period between 2nd March 1998 to 30th October 2011. The NSE 20-share index is a weighted mean with 1966 as the base year at 100. It is based on 20 companies calculated on a daily basis. The index is useful in determining the performance of the NSE by measuring the general price movement in the listed shares of the stock exchange. Bamburi Cement, Ltd. was founded in 1951 and manufactures cement in sub-Saharan Africa. The Kenya Airways' principal activities include passengers and cargo carriage. It was incorporated in 1977 as the East African Airways Corporation (EAA). The company was listed in the NSE in 1996 and has been a major player in the Nairobi stock market. The National Bank of Kenya Limited (NBK) was incorporated on 19th, June 1968 and officially opened on Thursday 14th, November 1968. Its main objective was to help Kenyans to get access to credit and control their economy after independence.

The preliminary analysis was done by use of time plots for the various series presented in Figures 1 and 2.

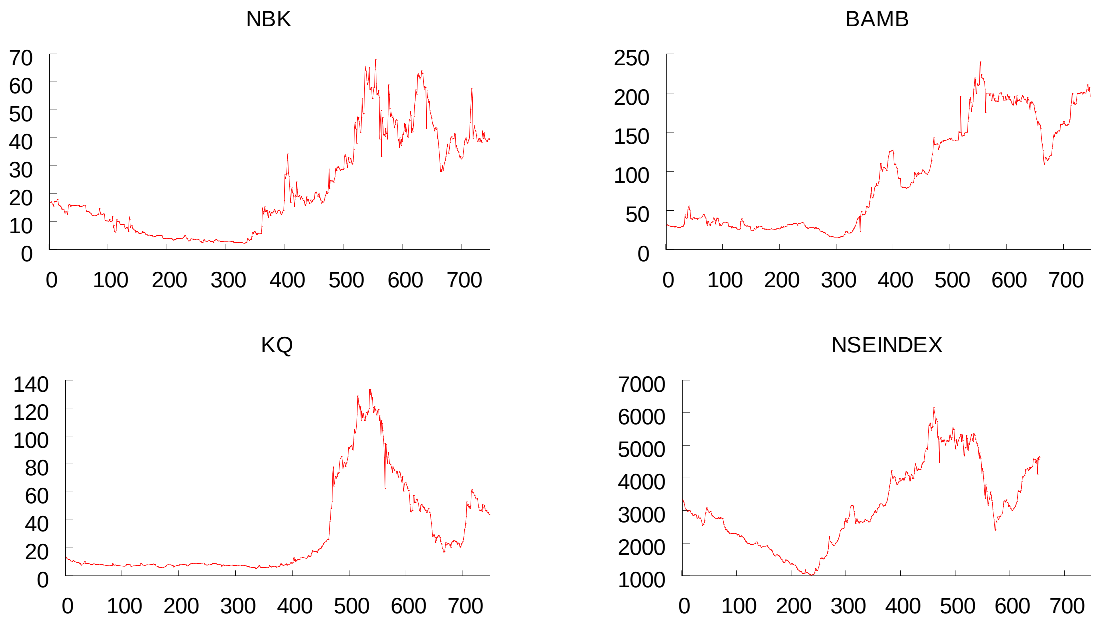

Figure 1: Time plots for the weekly average prices

A visual inspection of the time plots clearly shows that the mean and variance are not constant, implying non-stationarity of the data. The non-constant mean and variance suggests the utilization of a nonlinear model and preferably a non-normal distribution for modelling the data.

The series were transformed by taking the first differences of the natural logarithms of the values in each the series. The transformation was aimed at attaining stationarity in the first moment. The equation representing the transformation is given by $X_t = \ln(P_t) - \ln(P_{t-1})$,where P_t represents the weekly average value for each series. The sequence plots for the returns are presented in Figure 2.

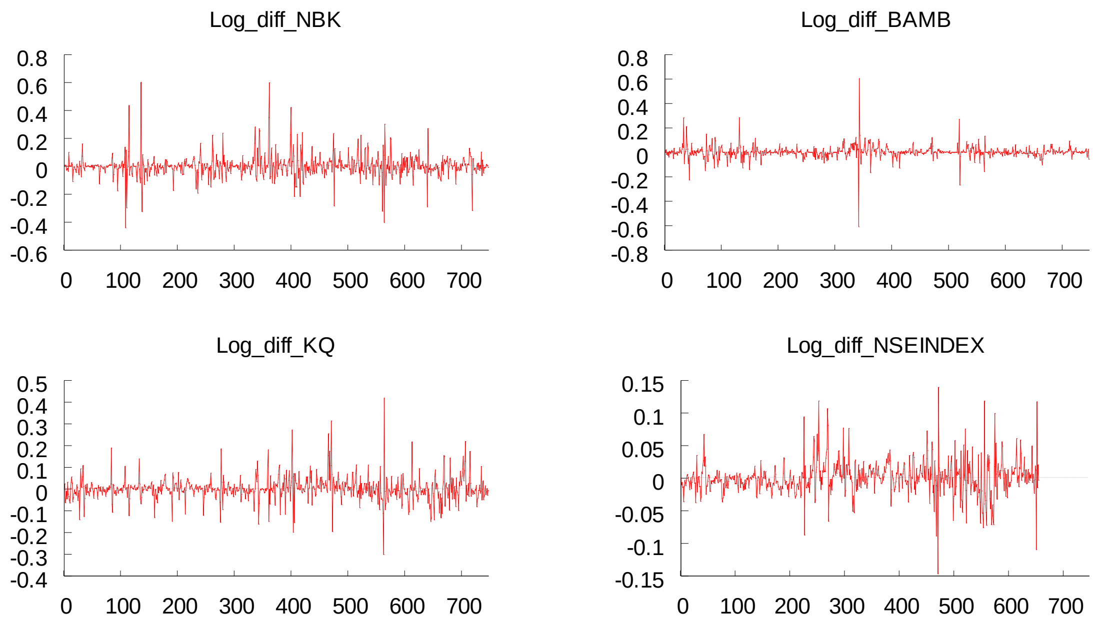

Figure 2: Time plots for log differenced series

The basic statistical properties of the data show that mean returns are all positive and close to zero a characteristic common in the financial return series. All the four series have very heavy tails showing a strong departure from the Gaussian assumption. The Jarque-Bera test also clearly rejects the null hypothesis of normality. Notable is the fact that all the four series exhibit positive Skewness estimate. This means that there are more observations on the right hand side.

The series having exhibited heteroscedasticity as shown by the time plots were tested for the ARCH disturbances using Engle's (1982) Lagrange Multiplier (LM) while the Portmanteau Q test (McLeod and Li, 1983) based on the squared residuals was used to test for the independence of the series. Since both the Q statistic and the LM are calculated from the squared residuals, they were used to identify the order of the ARCH process. For all the return series, the Q statistics and the Lagrange Multiplier (LM) tests indicated strong heteroscedasticity for all the lags from 1 to 12.This suggested an ARCH model of order q=8.

4.1 Empirical Results and Discussions

4.1.1 ARCH models

The first set of models implemented in this study was the original Engle's (1982) ARCH models. The student's t-distribution and the General Error Distribution (GED) were tested for all the series. The student's t distribution assumption provided a better model for NBK and KQ while the GED performed well for NSE Index and Bamburi. This could be due to the fact the financial data is highly heavy tailed and is better captured by the student's t-distribution since the GED distribution has a higher peak than the student's t-distribution. Although the GED distribution may be better able to capture peaks, it is far worse for capturing fat tails. The Jarque-Bera (1980) statistic also strongly rejected the normality assumption in the standardized residuals for all the series. The fitted models were adequate since their standardized residuals were not significantly correlated in all the four series basing on the Ljung-Box Q statistics. The squared residuals were also not significantly correlated for lags up to 12 for all the four series.

4.1.2 GARCH models

The next class of models implemented was the GARCH models. The autoregressive models were applied to capture the autocorrelation present in the series. The GARCH models for different values of p and q were fitted to the data, diagnosed and from the diagnosis and goodness of fit statistics, the GARCH (1,1) was found to be the best choice. This is consistent with most empirical studies involving the application of GARCH models in financial time series data. The Maximum Likelihood Estimation (MLE) method was employed in the parameter estimation.

The GARCH parameter estimates for the variance equation was significant for all the series except for the NSE Index in which α_1 was not statistically significant. In the GARCH model, the parameters α and β must satisfy $\alpha_1 + \beta_1 < 1$ for stationarity. However, the GARCH (1,1) estimates violated the restriction imposed i.e. in all cases, $\alpha_1 + \beta_1 > 1$. This implies that the fitted GARCH model is not weakly stationary and the conditional variance (σ_t^2) does not approach the unconditional variance (σ^2) and thus the series might not have finite unconditional variance. This calls for the implementation of Integrated GARCH (1,1) model since it is capable of being stationary in the strong sense even though $\alpha_1 + \beta_1 = 1$ (Nelson,1990).

Two distributions were tested (i.e student's t and GED) for the specific GARCH (p,q) model and the best distribution choice was determined based on the SBC, AIC and the Log likelihood Ratio test in all the cases (see Table 4.8). For the NSE index, the distribution of choice was the student's t-distribution while for NBK, Bamburi, and KQ the Generalized Error Distribution was chosen. This shows that the NSE index data had fatter tails as compared to NBK, Bamburi and KQ. The model adequacy was checked using the Ljung-Box Q statistics for residuals and squared residuals in which the null hypothesis of no significant correlations was not rejected for all the series implying that the fitted models were adequate. The JB statistics rejected the null hypothesis of normality in the standardized residuals. This implies that the models with the respective distributions failed to normalize the residuals. The Goodness of fit statistics and Diagnostic tests are presented Appendix 2.

4.1.3 Integrated GARCH (IGARCH) Model

Since the parameter estimates in GARCH (1,1) models were close to the unit root but not less than unit, i.e $\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j = 1$,the IGARCH model was fitted. The MLE method was utilized for parameter estimation for the mean and variance equations.

The parameter estimates for the variance equation were statistically significant at 0.05 significance level in all the series. In addition, $\alpha_1 + \beta_1 = 1$ for all the cases; implying that multi-step forecasts of the conditional variance do not approach the unconditional variance (i.e the unconditional variance is infinite). Despite the infinite unconditional variance, one attractive feature of the IGARCH model is that it is strongly stationary even though it is not weakly stationary. The results indicate that the data sets used exhibit the persistence in variance/volatility whereby the current information remains important in forecasting the conditional variance, i.e. the current information in the NSE remains important in forecasting the conditional variance.

Two distribution assumptions namely, Generalized error Distribution and t-distributions were tested. Generalized error Distribution provided the best fit for the data adequately when modelling with the IGARCH model in all the four series. The models were fitted and diagnosed using the AIC, SBC and the Log likelihood ratio test. However, the final model was considered adequate if its standardized residuals and squared residuals were not significantly correlated at 5% significance level. The residual correlation was tested using the Ljung-Box Q statistics. All the fitted IGARCH models were adequate since their residuals were not significantly correlated. Further, the standardized residuals were still non-normal as shown by the JB statistics for normality. The goodness of fit statistics for the IGARCH(1,1) model and the diagnostic tests are presented in Appendix 1 and 2 respectively. In order to capture the leverage effects, two asymmetric ARCH-type models; the Exponential GARCH (EGARCH) and Threshold GARCH (TGARCH) were fitted.

4.1.4 EGARCH models

Despite the popularity and apparent success of GARCH models in practical applications, they cannot capture asymmetric response of volatility to news since the sign of the returns play no role in the model specification. Statistically, the asymmetric effect occurs when an unexpected decrease in price resulting from bad news increases volatility more than an unexpected increase in price of similar magnitude following good news.

Accordingly, Nelson's (1991) EGARCH model was fitted. Unlike the GARCH (p,q) model, a negative shock can have a different impact on future volatility when compared to the positive shock if asymmetry parameter γ_1 is not zero for the EGARCH model. It also does not need restrictions to be imposed on the parameters to ensure the non-negativity.

In the EGARCH model estimation, the MLE criterion was employed. Different orders for p and q in the variance equation were tested with the best results being achieved for p=q=1. The Generalized Error Distribution emerged as the best distribution for all the series (NSE, Bamburi, KQ and NBK). This implies that all the series under investigation have long tails and are asymmetric.

The EGARCH model parameter estimates also reveal the persistence in volatility of the Nairobi equity market. This is because the sum of α_1 and β_1 is approximately 1 in all the data sets. The asymmetric parameter γ_1 is positive and significant for all the four series namely NSE Index,NBK, Bamburi and KQ. The positivity of γ_1 indicates that positive shocks increase volatility more than the negative shocks of an equal magnitude. This shows that the concept of "leverage effect" (i.e the negative shocks increasing volatility more than a positive shock of the same magnitude) is not applicable to the individual company stocks. This is consistent with the earlier studies on the Nairobi Stock Exchange for instance Ogum *et al.*, (2005, 2006) who found the asymmetry parameter γ_1 to be positive when modelling the daily NSE 20 Share Index using the EGARCH models.

This could arise from the fact that the weekly return series were used in this study while Ogum *et al.*, (2005, 2006) modelled the daily returns. Some information could have been lost when using the weekly average for the NSE index and the share prices for the companies. In addition, the flow of information in NSE might not be as efficient as in the developed equity markets.

The model diagnostics and goodness of fit statistics are presented in Appendix 1 and 2 respectively. The diagnostics included the autocorrelation of the standardized residuals and squared residuals respectively. The Ljung-Box Q statistics represented by Q(12) and $Q^2(12)$ for residuals and squared residuals respectively were used which were not significant in all cases confirming the adequacy of the fitted models. The models could thus explain the non-linear dependence in the residuals i.e the models captured the dependence in the variance shown by the original series of returns. The EGARCH model, in all cases showed a smaller Kurtosis compared to the ARCH and GARCH models. In addition, the student's t-distribution and Generalized Error Distributions also captured the tail properties of the data better than the Gaussian distribution in all the four cases. The JB statistics also strongly rejected the null hypothesis of normality in the standardized residuals in all the series under consideration.

4.1.5 Threshold GARCH (1,1)

The TGARCH (1,1) model which falls in the asymmetric class of ARCH-type models was also used. The model was fitted, estimated and diagnosed just like the previous models. From the two distributions tested, the student's t distribution emerged the best for the NSE index while GED was considered the best for the NBK, Bamburi and KQ. This is because the GED and the students's t-distributions were able to capture the tail properties of the data. It is worth noting that under the student's t distribution, the convergence during estimation was a major problem. The algorithm converged very slowly and sometimes weakly. This casts doubts on the stability of the parameter estimates.

In the variance equation, the asymmetry parameter γ_1 was less than zero for all the four series. This implies that good news increases volatility more than bad news. This is consistent with the findings of Ogum *et al.*, (2005, 2006) who applied EGARCH models to the daily NSE 20 Share Index. Hence the leverage effect experienced in developed markets might not be a universal phenomenon.

The diagnostic tests and goodness of fit statistics for the TGARCH models are presented in Appendix 1 and 2 respectively. Just like the previous models, the best distributions were GED and the student's t-distribution.

Also, based on the Ljung-Box Q statistics, both the residuals and the squared residuals were not significantly (5% level) correlated implying that the models were adequate. The JB statistic for normality also rejected the normality assumption in the standardized residuals.

4.2 Efficiency Comparison between the ARCH-type Models

Model efficiencies for each of the ARCH-type models implemented were evaluated using the various MSE. The MSE for the chosen models are presented in Table 1.

Table 1: MSE for the fitted ARCH-type models

Series	ARCH(q)	GARCH(1,1)	IGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)
NSE INDEX	0.0006946	0.0006958	0.000696	0.0006967	0.000699
NBK	0.006757	0.006765	0.006744	0.006766	0.006765
BAMBURI	0.003119	0.003073	0.003098	0.003019	0.003073
KQ	0.002997	0.002993	0.002989	0.002996	0.002993

Considering the MSE values in Table1, it is clear that ARCH, GARCH, EGARCH and IGARCH are all equally efficient in modelling volatility based on the MSEs only, since the different ARCH-type models are almost equal for the respective data sets. The disadvantage with the ARCH model is that so many parameters are to be estimated. The GARCH, IGARCH, EGARCH and TGARCH models are able to parsimoniously model the series and hence are preferred to the original ARCH model. Considering the asymmetric properties of the data and the respective MSEs, the EGARCH (1,1) emerged as the best model for the NSE Index and Bamburi. For the NBK, both the EGARCH and the TGARCH are equally good but EGARCH is considered the best since the parameter estimates for the TGARCH are unstable due to weak convergence. The best model for Kenya Airways was the GARCH model.

The respective models chosen are justified by their relatively lower values of residual Kurtosis and MSE in addition to the other diagnostics considered as well as the asymmetric parameter that captures the leverage effect. However, in terms of stationarity, the IGARCH model with the Generalized Error Distribution (GED) emerged as the best ARCH-type model since it was strongly stationary thus being more stable. This makes the IGARCH model to be the preferred model from the ARCH-type models for modelling the Nairobi Stock Exchange data for the periods between 2nd March 1998 to 30th October 2010 for NSE 20-Share index while and 3rd June 1996 to 30th October 2010 for company share prices, i.e NBK, Bamburi and Kenya Airways.

5.0 Summary and Conclusions

In this study, the original Engle's (1982) ARCH (p) model and its three extensions namely, standard GARCH (p,q), IGARCH(p,q), EGARCH (p,q) and TGARCH (p,q) were applied to the data. Different orders for ARCH(p) were tested in all cases where p=8 was found to be the most adequate for NSE index, Bamburi and KQ while for the NBK series, p=9 provided the best order for ARCH model. Four different p and q values were tested for GARCH(p,q), EGARCH (p,q) and TGARCH (p,q): (1,1), (1,2), (2,1) and (2,2). The order p, q equal to (1,1) is by far the most used values in GARCH research today and results obtained is also consistent with this.

In all the four series, the order (1,1) is the best choice. Comparing the diagnostics and the goodness of fit statistics, the IGARCH (1,1) outperformed the ARCH, EGARCH and TGARCH models majorly due to its stationarity in the strong sense. However, the IGARCH model is unable to capture the asymmetry exhibited by the stock data. The EGARCH (1,1) and the TGARCH (1,1) are the preferred models to describe the dependence in variance for all the four series studied since they were able to model asymmetry and parsimoniously represent a higher order ARCH(p). However, the standardized residuals still displayed non-normality in all cases.

Judging from the asymmetric parameter ($\gamma_1 < 0$) in the EGARCH model, the volatility increases more with the bad news (negative shocks) than the good news (positive shocks) of the same magnitude for the NSE Index. This is not consistent with the findings of Ogum *et al.*, (2005, 2006). However, for the individual stocks the asymmetric parameter ($\gamma_1 > 0$) meaning that volatility increases more for good news more than bad news of the same magnitude. This implies that the leverage effect may not be a universal phenomenon after all. From the different distributions tested and estimated, the student's t distribution was the best choice for NSE index while GED was the best for NBK, Bamburi and Kenya Airways. The Gaussian assumption provided the poorest results and in some cases had convergence failures.

6.0 References

Akaike, H. (1974). A new look at the statistical model identification. *IEE. Autom. control* **19**, 716-723.

Andrew,W and Whitney, K. (1986). A large sample chow for the single linear simultaneous equation. *Economic letters* **19**,351-353.

Appiah-Kusi, J. and Pescetto, G. (1998). Volatility and Volatility Spill-Overs in Emerging Markets, CERF Discussion Paper Series No. 98-04.

Baudouhat, A.V. (2004). *Nordic Financial Market Integration:An Analysis with GARCH.* Unpublished Msc Thesis, Göteborg University

Bera, A.K. and Higgins, M.L. (1993). On ARCH models: Properties, Estimation and Testing, *Journal of Economic Surveys*, **7**, 305-366.

Berndt, E.K, Hall, B.H., Hall, R.E and Hausman, J. (1974). Estimation and Inference in Nonlinear Structural Models. *Annals of Economic and Social Measurement*, **3**,653-665.

Bertram, W. (2004). *An empirical investigation of Australian Stock Exchange Data*. School of Mathematics and Statistics, University of Sydney.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroscedasticity. *Journal of econometrics*,**31**,307-327.

Brooks, R. D., Davidson, S. and Faff, R. W. (1997). An Examination of the Effects of Major Political Change on Stock Market Volatility: The South African Experience, *Journal of International Financial Markets, Institutions & Money*,**7**, 255 - 275.

Curto, J.J.D.(2002). *Modelling the volatility in the Portuguese Stock Market: A Comparative Study with German and US markets*. Unpublished Phd Thesis, Lisbon University.

Degiannakis, S. and Xekalaki, E. (2004). Autoregressive Conditional Heteroscedasticity Models: A Review. *Quality Technology and Quantitative Management*, **1(2),** 271-324.

Engle, R.F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K inflation, *Econometrica*, **50**, 987-1008.

Engle, R.F. (1983). Estimates of the Variance of US Inflation Based on the ARCH Model. *Journal of Money Credit and Banking*, **15**, 286–301.

Engle, R.F.,Lilien, D.and Robins,R. (1987). Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M model, *Econometrica*, **55 (2),** 391-407.

Engle, R.F. (2004). Risk and Volatility: Econometric Models and Financial Practice. *The American Economic Review*, **94**, 405-420.

Fama, E, Fisher, L., Jensen, M., Rolls, R. (1969). The adjustment of stock prices to newinformation. *International Economic Review*, **69**, 1-21.

Fama, E. (1965). The behaviour of stock market prices. *J.Business,* **38**, 34-104.

Fan, J. and Yao, Q.(2003). *Nonlinear Time Series: Nonparametric methods and Parametric methods.* New York: Springer Series in Statistics.

Frimpong, J.M and Oteng-Abayie, E.F.(2006). Modelling and Forecasting Volatility of Returns on the Ghana Stock Exchange Using Garch Models. *American Journal of Applied Sciences*, **3 (10)**, 2042-2048.

Gary, T and Mingyuan, G. (2004). *Intraday Data and Volatility Models: Evidence from Chinese Stocks.* University of Western Sydney.

Glosten, L.,Jagannathan, R, and Runkle, D. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, **48**, 1779-1801.

Hongyu, P and Zhichao, Z.(2006). Forecasting Financial Volatility: Evidence from Chinese Stock Market. Working paper in economics and finance No. 06/02. University of Durham. United Kingdom.

Mandelbrot,B. (1963). The variation of speculative prices. *J. Bus*, **36**, 394-419.

McLeod, A.I. and Li, W.K. (1983). Diagnostic checking ARMA Time Series Models Using Squared-Residual Autocorrelations, *Journal of Time Series Analysis*, **4**, 269-273.

Mills, T.C. (1999). *The econometric Modelling of Financial Time Series*, Cambridge Univ.Press.

Nelson, D. (1990). Stationarity and persistence in the GARCH(1,1) models. *Econometric Theory* **6**, 318-334.

Nelson, D.B. (1991): Conditional Heteroskedasticity in Asset Returns: a New Approach. *Econometrica,* **59.2,** 347-370.

Ogum, G., Beer, F. and Nouyrigat, G. (2006). An Empirical Analysis of the Kenyan and the Nigerian daily returns using EGARCH Models. In http://www.euromedia-marseille.com/ media/Nouyrigat_-_ISINI.PDF

Sentana, E. (1995). Quadratic ARCH models. *Review of Economic Studies*. **62 (4)**, 639-661.

Poon, S.H. and C. Granger (2003). Forecasting volatility in financial markets: a review, *Journal of Economic Literature*, **XLI**, 478-539.

Priestly, M.B. (1988). *Non-Linear and Non-Stationary Time Series Analysis*, Academic Press, New York

Schwarz, G. (1978).Estimating the Dimension of a Model. *Ann. Inst.statistic. Math.***6**,461-464.

Walter,A.D. (2005). A structural GARCH model: An application to portfolio risk management Unpublished PhD Thesis, University of Pretoria.

Zakoian, J-M. (1994). Threshold Heteroskedastic Models. *Journal of Economic Dynamics and Control.* **18**, 931-955.

APPENDICES

Appendix 1: The goodness of fit statistics for ARCH models

		ARCH (q)		GARCH(1,1)		IGARCH(1,1)		EGARCH(1,1)		TGARCH	
		t	GED	t	GED	t	GED	t	GED	t	GED
NSE INDEX	LR	1641.062	1641.492	1639.411	1638.036	1618.032	1619.657	1641.107	1645.863	1641.85	1638.305
	AIC	-5.012538	-5.013531	-5.025961	-5.021717	-4.966148	-4.971165	-5.02811	-5.04279	-5.0304	-5.019461
	SBC	-4.895168	-4.896161	-4.996500	-4.945771	-4.904010	-4.909027	-4.94526	-4.95994	-4.9476	-4.93661
NBK	LR	1089.142	1083.861	1098.053	1106.464	1052.383	1070.120	1106.751	1112.412	1098.55	1106.873
	AIC	-2.891364	-2.877150	-2.934195	-2.956834	-2.816643	-2.864387	-2.95492	-2.9702	-2.9328	-2.955242
	SBC	-2.798281	-2.784068	-2.884551	-2.907190	-2.779410	-2.827154	-2.89906	-2.91430	-2.8769	-2.899393
BAMBURI	LR	1350.889	1464.813	1529.121	1554.337	1484.791	1524.654	1533.434	1561.069	1530.18	1555.099
	AIC	-3.602938	-3.910428	-4.100193	-4.168251	-3.985940	-4.093532	-4.1091	-4.1837	-4.1004	-4.167610
	SBC	-3.503440	-3.810930	-4.038007	-4.106065	-3.936191	-4.043783	-4.04073	-4.11532	-4.032	-4.099205
KQ	LR	1349.801	1269.075	1353.618	1360.433	1306.834	1323.664	1351.000	1357.335	1355.10	1361.186
	AIC	-3.595151	-3.377561	-3.624307	-3.642678	-3.503596	-3.548960	-3.61456	-3.63163	-3.6256	-3.642011
	SBC	-3.495758	-3.278168	-3.568399	-3.586769	-3.460112	-3.505475	-3.55243	-3.56951	-3.5635	-3.579891

LR- Represents Log likelihood Ratio test
JB- Represents Jarque-Bera statistics for normality
Q(12) - Represents Ljung-Box Q statistics for the standardized residuals
$Q^2(12)$ - Represents Ljung-Box Q statistics for squared standardized residuals
P-Values are given in the brackets

Appendix 2: Diagnostic Tests for Standardized Residuals for ARCH-type models

Series	Statistics	ARCH(q)	GARCH(1,1)	IGARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)
NSE INDEX	Skewness	0.371885	0.453360	0.274917	0.604607	0.510931
	Kurtosis	10.74517	10.92675	9.247141	9.579835	10.59302
	JB	1634.604 (0.00)	1718.698 (0.00)	1061.89(0.000)	1208.424 (0.00)	1584.85 (0.000)
	Q(12)	6.7497 (0.345)	7.5418 (0.274)	5.2777 (0.509)	7.4763 (0.279)	7.0248 (0.319)
	$Q^2(12)$	6.7934 (0.340)	5.7669 (0.450)	13.047 (0.560)	5.2634 (0.511)	6.3818 (0.382)
NBK	Skewness	0.439535	0.280164	-0.228243	0.249078	0.247907
	Kurtosis	12.07521	13.12185	22.04193	12.30715	13.08784
	JB	2573 (0.000)	3181.45(0.000)	11231.8 (0.000)	2689.39 (0.000)	3158.07 (0.000)
	Q(12)	10.752(0.293)	10.642 (0.301)	16.739 (0.053)	11.305 (0.255)	10.215 (0.333)
	$Q^2(12)$	6.7901 (0.659)	3.4587 (0.943)	47.096 (0.000)	2.7684 (0.973)	3.2014 (0.956)
BAMBURI	Skewness	-6.437273	-1.483386	1.232353	-1.523194	-1.420842
	Kurtosis	128.6041	34.14905	28.68903	28.60294	30.41023
	JB	492213.6 (0.00)	30228.6(0.000)	20562.8 (0.000)	22137.3 (0.000)	23446.36 (0.00)
	Q(12)	10.311 (0.172)	13.569 (0.06)	18.593 (0.670)	16.906 (0.180)	13.770 (0.06)
	$Q^2(12)$	0.1257 (1.000)	1.1426 (0.992)	18.593 (0.100)	1.9102 (0.965)	1.4412 (0.984)
KQ	Skewness	0.578515	0.729736	0.615099	0.558778	0.626258
	Kurtosis	8.830365	10.65188	11.20788	9.962207	10.49333
	JB	1092.344 (0.00)	1876.064 (0.00)	2129.62 (0.00)	1537.216 (0.00)	1784.475 (0.00)
	Q(12)	14.402 (0.072)	13.200(0.105)	5.8447 (0.665)	10.898 (0.208)	12.370 (0.135)
	$Q^2(12)$	6.6472	6.0670 (0.640)	69.101 (0.450)	6.1931 (0.626)	5.8701 (0.662)