Mathematical Logic Knowledge of Science Teacher Candidates in Newton’s Laws of Motion

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Abstract
In this research, status of knowledge of science teacher candidates in Newton’s laws of motion was determined through their achievement level, knowledge level and the factors that are likely to have impact on their achievement levels using qualitative and quantitative methods. In the quantitative stage, the achievement level of teacher candidates was found to be 15%; whereas it was found to be 39% in the qualitative stage. Since teacher candidates’ knowledge level was based only on the variable of operation in which mathematical knowledge is used, it was concluded that they know only the mathematical dimension of formula or laws of nature. It could then be argued that it would be unlikely to take candidates’ knowledge beyond simply memorizing without teaching them logical structures of subjects.

Keywords: Mathematical logic, students’ knowledge and achievement levels, variables in achievement, factors in achievement.

1. Introduction
The history of modern logic is included within the history of math or, more generally, symbolic logic. Modern logic is also called “new logic”, “mathematical logic” or “symbolic logic”. In fact, it is mathematicians who founded the modern logic. The pioneer of this logic is George Boole. The ideas presented by him are related to the algebraic expression of the laws of thought. Not only Gottlob Frege but also Hermann Rudolf Lotze and Ernst Schröder stated similar opinions to the “algebraic expression of the logic”, an idea presented by Boole (Peckhaus, 1999).

Although logic and math started approaching each other with Newton’s and Leibnitz’s differential calculus and integral calculus- it was Boole and Frege who combined the two. Boole and Frege made attempts to provide a final and clear from to what formal deduction is, which, in turn, resulted in math and logic moving nearer to each other. Boole developed a symbolic system for Aristotle’s rules of deduction. Even though Aristotle had articulated his rules of deduction in a clear way, they were in natural language (verbally). However, Boole attempted and managed to develop a symbolic system by extending them beyond this kind of an expression. Frege, on the other hand, improved on Boole’s “symbolic system for rules of deduction”, came up with “predicate calculus (open-ended propositions)”, and laid the foundations for mathematical logic (Özenli, 1999, p: R1; Heijenoort, 1970, pp: 1-2; Corcoran, 2003). Currently, this calculus constitutes a sufficient part of the logical basis of the whole math (Özenli, 1999, pp: R1-R2). Mathematical logic is not a formulated abstract logic, but it expresses the content through symbols in a more definitive and decisive way than words do by Frege (Gözkan, 2008, p: 20).

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George Boole found that, by adequately representing Logic, it became a branch of Algebra in a precise sense: all known results in Logic (and some unknown) could be obtained by the use of standard mathematical techniques. Two facts were crucial for Boole’s discovery:

(i) First, the change in representation from a philosophical view of Logic as an enormous complex structure of syllogisms and conditional statements, to a view of it as (possibly) fitting the general scheme for any branch of Algebra. Simplistically, the scheme consists of a set of very simple objects that are subjected to operations on numbers. (ii) The second fact was the successful application of the MSS to Logic, which meant that some (interesting) subset of properties of the operations on numbers held also for Logic. This made it possible to use the powerful tools of Mathematics on this new domain of Logic (Ledesma, et al., 1997).

Frege, also wished to apply it to making reasoning about human affairs more rigorous. Indeed, Leibniz was explicit about his goal of replacing argument with calculation. However, expressing knowledge and reasoning about the commonsense world in mathematical logic has entailed difficulties that seem to require extensions of the basic concepts of logic, and these extensions are only beginning to develop (McCarthy, 1988).


2. Material and Method

The data for the study were collected from 1st grade prospective science teachers taking the lesson Physics 1, in which Newton’s laws of motion are covered, through quantitative and qualitative case studies. “Holistic multi-state design” and “holistic single-state design” were used for the quantitative and qualitative parts of the study respectively. The quantitative data were collected through a 7-item questionnaire designed with the purpose of collecting personal information about students and a test which consisted of 11 multiple-choice questions on mathematical logic knowledge, whose reliability and validity had been established beforehand. The questionnaire on personal information included questions as to a) gender, b) university, c) type of high school d) time spent on studying Newton’s laws of motion e) styles of studying Newton’s laws of motion f) achievement score in general physics 1 g) achievement score in general math 1. Since propositions are expressed in mathematical logic through mathematical symbols (Cocorcan, 2003; Ledesma, et al., 1997), teacher candidates were given propositions in the questions of the assessment instrument and asked if physics formulae complied with these propositions. These questions were formulated by the researcher. Mathematical logic knowledge questions cover the subjects of Kepler’s laws, work-energy, linear momentum, potential energy, power, kinetic energy, gravitational force, spring force and centripetal motion.

The qualitative data for the study were collected through three different measurement tools. The first one is “The Qualitative Measurement Tool 1 (the QMT1) which is comprised of six semi-structured questions. Four questions were chosen among the questions used for the test on mathematical logic knowledge. Two questions were formulated by the researcher. The second one, “The Qualitative Measurement Tool (the QMT2)”, includes the procedures for the QMT1. The QMT2 is comprised of 36 semi-structured questions to measure whether students have a clear idea about the procedures for the QMT1. The third measurement tool, “The Qualitative Measurement Tool 3 (the QMT3)”, contains 50 semi-structured questions to measure their basic knowledge about math which is required for the questions in the QMT1. Forty-one of these questions were borrowed from the literature (Haeussler&Paul, 1993; Karakaş, 2001, p: 100). The remaining 9 questions were composed by the researcher.

The population of the study was comprised of 1st grade prospective science students from the faculties of education located in Turkey. The sample of the study from whom the quantitative data were collected was comprised of 599 1st grade students studying at the department of Science Teaching, faculty of education, at seven universities in Turkey, during the second term of the educational year 2009-2010. These students had already taken general physics 1 and general math 1. The universities were selected on the basis of provincial achievement and achievement of the universities themselves. The sample of the study from whom the qualitative data were collected was comprised of 7 voluntary students who had answered the questions included within the quantitative data collection tools and whose answers differed in achievement score in general physics 1, achievement score in general math 1 and answers to the questions in the questionnaire.
The present study was implemented in two stages, namely the quantitative one and the qualitative one. At first, the data for the study were collected through the questionnaire on personal information and test on mathematical logic knowledge questions (the quantitative stage). Afterwards, the QMT1, QMT2 and QMT3 were implemented (the qualitative stage). Through these measurement tools, the data were collected in one session with two parts, namely the written one and the interview-based one. During the former, the students were informed about the measurement tools. After that, they were asked to solve the problems in the QMT2, QMT3 and the QMT1 respectively. During the second part, the data were collected through interviews in which the students were asked to explain how they solved the problems in the QMT1. When suitable, their answers were compared with the data from the QMT2 and QMT3 and they were addressed with certain questions. Furthermore, when the need arose, the students were asked to explain some of the data obtained from the QMT1.

The status of students’ mathematical logic knowledge in Newton’s laws of motion was determined through their achievement level, knowledge level and the factors that are likely to have impact on their achievement levels. Students’ achievement levels, on the other hand, were determined through quantitative and qualitative stages. Their achievement levels were discovered through the test on mathematical logic knowledge and the QMT1 in the quantitative stage and qualitative stage respectively. Their achievement in the qualitative stage was discovered through the written part. The correct answers provided by the students to the questions included in the test on mathematical logic knowledge were regarded as “The ratio of correct answers to the test in Table 1” whereas the correct answers they provided to the questions included in the QMT1 were accepted as “The ASS Value by Percentage”. Their achievement levels in the QMT2 and QMT3 were determined as well.

The reasons that might have an effect on their achievement score were divided into two groups, namely variables and factors in achievement. In this study, the factors in achievement were determined through the quantitative and qualitative stages. Some of the factors measured through the quantitative stage cannot be changed. Even so, they can be improved, which is actually one of the objectives of education. On the other hand, the factors measured through the qualitative stage can be changed.

Some of these factors were measured with the items included in the questionnaire on personal information. The other factors in achievement were measured through the QMT2 and QMT3 during the written part of the qualitative stage. An attempt was made to determine whether there was a significant correlation between the items included in the questionnaire on personal information and the test on mathematical logic knowledge. The factors between which there was a significant correlation are those factors which have an influence on achievement. On the other hand, the factors between which there was no significant correlation are factors which do not have an influence on achievement. A calculation was made into the effect of the factors “the QMT1” and “QMT2”, which were measured in the qualitative stage, on the result (ASS) by percentage, i.e. on achievement level.

In this study, the variables in achievement were measured through the written and interview-based parts in the qualitative stage. The variables in achievement are as follows: a) given-asked b) definitions for solutions to the questions c) formulas for solutions to the questions d) operations required for solutions to the questions. The students’ scores in these variables were calculated in order to determine the effect of the variables on the score in the QMT1 (ASS). Their mathematical logic knowledge about Newton’s laws of motion was determined with a consideration into the fact that their scores in the variables might have an effect on the result (ASS).

An interval scale was used for their achievement scores in general physics 1 and general math 1. A nominal scale was used for their gender, university, type of high school, time spent on studying Newton’s laws of motion, styles of studying Newton’s laws of motion, the test on mathematical logic knowledge, the QMT1, QMT2 and the QMT3.

SPSS was used for the analysis of the correlation between the answers to the questionnaire on personal information and the test on mathematical logic knowledge. The data obtained through the QMT1, QMT2 and the QMT3 were analyzed through a software program developed for Probability and Possibility Calculation Statistics for Data Variables (VDOIH1); Statistical Methods for Combined Stage Percentage Calculation (Yılmaz, 2011; Yılmaz&Yalçın, 2011). Statistical methods for combined stage percentage calculation determines the values of variables and makes an analysis of the effect of these variables on the result.

3 This study considers the QMT 2 and QMT 3, which were measured in the qualitative stage, as factors in achievement. They can also be accepted as variables in different studies.
In this method, the variable to be measured is divided into stages, which, in turn, is divided into the smallest meaningful pieces. Afterwards, they are scored. The smallest meaningful pieces are scored as -1, 0 and 1. Furthermore, the stages of a variable are divided into three stages, namely positive, negative and unconnected stages. Afterwards, an attempt is made to analyze the effects of these stages on the result by percentage. The smallest meaningful pieces of the same stages of a variable are combined and turned into one single stage (Yılmaz & Yalçın, 2011).

3. Conclusion and Interpretations

Students’ achievement level in the quantitative stage of the research, which is linked to the status of their knowledge in Newton’s laws of motion, was determined on the basis of the responses given by 599 participant students to the questions in the mathematical logic knowledge test on Newton’s laws of motion. At the quantitative stage, students’ achievement level was found to be 15%. That is, the students who participated in the quantitative stage scored 988 out of 6589 points from eleven mathematical logic questions. Thus, students’ achievement in the mathematical logic knowledge test is ~0.15. The results obtained through assessment instruments are given in Table 1.

The achievement level of the students who participated in the qualitative stage of the research was determined based on their responses to the questions in QMT1. The achievement level in the qualitative stage was determined through written applications. The achievement level of seven students who participated in written applications in mathematical logic knowledge questions related to Newton’s laws of motion was found to be 39%. That is, the participant students scored 16.38 out of 42 points with the responses they gave to six mathematical logic knowledge questions. In QMT2 and QMT3, which are other assessment instruments employed in the written application of the qualitative stage, students’ achievement levels were found to be 54% and 82%, respectively.

The following are the findings related to the factors that were likely to have impact on the achievement scores of students related to the status of their mathematical logic knowledge on Newton’s laws of motion (based on the total scores obtained from the questions of the mathematical logic knowledge test administered in the quantitative stage): a) It was determined, via the analysis of t-test with respect to students’ genders, that male students scored significantly higher than female students. b) A significant difference was found in the ANOVA analysis with respect to their universities. To determine between which universities this significant difference exists, Scheffe test was performed.

According to Scheffe test results, students attending University I when compared to those attending University V and University VI; students attending University II when compared to those attending University V; students attending University III when compared to those attending University V and University VI; students attending University IV when compared to those attending University V and University VI; and students attending University VII when compared to those attending University V received significantly higher scores from mathematical logic knowledge questions. c) No significant difference was found in the ANOVA analysis with respect to the types of high school from which students had graduated. d) No significant difference was found in the ANOVA analysis with respect to the times students spent studying “Newton’s laws of motion”. e) No significant difference was found in the ANOVA analysis with respect to the styles of studying “Newton’s laws of motion”. f) No significant correlation was found in the correlation analysis with respect to the passing grades from the General Physics I course and g) No significant correlation was found in the correlation analysis with respect to the passing grades from the General Mathematics I course.

In the quantitative stage; gender and university were found to be the factors influencing students’ achievement scores regarding the status of their mathematical logic knowledge. On the other hand, the factors that were found to be affectless are duration and style of studying “Newton’s laws of motion”, and grades from General Physics I and General Mathematics I courses. The factors of QMT2 and QMT3, which were determined at the qualitative stage, were interpreted with the findings obtained from the written application. It is thought that students’ QMT2 achievement level affected QMT1 achievement level by “ASS” 54%; and QMT3 achievement level affected QMT1 achievement level by “ASS” 82%.

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4 Significance analysis of the results was made at the level of p<.05.
5 Significance analysis of the results was made at the level of p<.01.
6 Significance analysis of the results was made at the level of p<.01.
The effects of the variables measured through the written part on the result “ASS” are as follows:

It is thought that the students’ knowledge in the positive stages of the variable “given-asked” has an effect of 3% on the ASS value. Their unconnected knowledge cannot affect the ASS value (0%). Similarly, their negative knowledge cannot have an influence on the ASS value (0%). Their positive knowledge in negative stages cannot have an influence on the ASS value (0%). It is thought that zero score has an effect of 97% on the ASS value.

It is thought that the students’ knowledge in the positive stages of the variable “definition” has an effect of 4% on the ASS value. Their unconnected knowledge cannot affect the ASS value (0%). Similarly, their negative knowledge cannot have an influence on the ASS value (0%). Their positive knowledge in negative stages cannot have an influence on the ASS value (0%). It is thought that zero score has an effect of 96% on the ASS value.

Since the students had no positive knowledge about the variable “formula”, the ASS score was not affected by this variable.

It is thought that the students’ knowledge in the positive stages of the variable “operation” has an effect of 36% on the ASS value. Their unconnected knowledge is thought to affect the ASS value negatively by 44%. Their negative knowledge is thought to affect the ASS value negatively by 14%. Their positive knowledge in the negative stages might have an influence of 3% on the ASS value. It is thought that zero score has an effect of 44% on the ASS value.

The collective effects of the four variables of the questions in the QMT1 on the result are as follows: Their knowledge in the positive stages has an effect of 14% on the ASS value. Their unconnected knowledge is thought to affect the ASS value negatively by 21%. Their negative knowledge is thought to affect the ASS value negatively by 5%. Their positive knowledge in the negative stages might have an influence of 3% on the ASS value. It is thought that zero score has an effect of 72% on the ASS value. Their knowledge about the QMT2 is thought to have an effect of 54% on the ASS value whereas their knowledge about the QMT3 is believed to have an effect of 82% on the ASS value.

The effects of the variables measured through the interview-based part on the result “ASS” are as follows:

It is thought that the students’ knowledge in the positive stages of the variable “given-asked” has an effect of 3% on the ASS value. Their unconnected knowledge cannot affect the ASS value (0%). Similarly, their negative knowledge cannot have an influence on the ASS value (0%). Their positive knowledge in negative stages cannot have an influence on the ASS value (0%). It is thought that zero score has an effect of 97% on the ASS value.

It is thought that the students’ knowledge in the positive stages of the variable “definition” has an effect of 13% on the ASS value. Their unconnected knowledge is thought to affect the ASS value negatively by 8%. Their negative knowledge is thought to affect the ASS value negatively by 10%. Their positive knowledge in the negative stages cannot have an influence on the ASS value (0%). It is thought that zero score has an effect of 77% on the ASS value.

It is thought that the students’ knowledge in the positive stages of the variable “formula” has an effect of 5% on the ASS value. Their unconnected knowledge cannot affect the ASS value (0%). Similarly, their negative knowledge cannot have an influence on the ASS value (0%). Their positive knowledge in the negative stages cannot have an influence on the ASS value (0%). It is thought that zero score has an effect of 95% on the ASS value.

It is thought that the students’ knowledge in the positive stages of the variable “operation” has an effect of 36% on the ASS value. Their unconnected knowledge is thought to affect the ASS value negatively by 44%. Their negative knowledge is thought to affect the ASS value negatively by 14%. Their positive knowledge in the negative stages might have an influence of 3% on the ASS value. It is thought that zero score has an effect of 44% on the ASS value.

The collective effects of the four variables of the questions in the QMT1 on the result are as follows: Their knowledge in the positive stages has an effect of 16% on the ASS value. Their unconnected knowledge is thought to affect the ASS value negatively by 22%. Their negative knowledge is thought to affect the ASS value negatively by 6%.
Their positive knowledge in the negative stages might have an influence of 1% on the ASS value. It is thought that zero score has an effect of 70% on the ASS value. Their knowledge about the QMT2 is thought to have an effect of 54% on the ASS value whereas their knowledge about the QMT3 is believed to have an effect of 82% on the ASS value.

4. Discussion and Suggestions

The fact that the first three variables in Table 1 APS values (knowledge values) of zero or close to zero shows that students failed to understand the data. The first condition of understanding data is the maximization of the knowledge level on the variable of “given-asked. After that, it should be taught that data do not only have a mathematical dimension. Results in Table 1 demonstrate that the knowledge levels of the variables of definition and formula should be maximized while teaching that a subject has a mathematical logic structure as well as a mathematical one. Findings of this research inform us about where to start and how to proceed in mathematical logic education. Undoubtedly, the first task is to maximize the knowledge level of the variable of given-asked. Then, knowledge levels of other variables should be maximized and IS values should be minimized.

Moreover, it should be targeted to maximize especially the knowledge level measured through QMT2. This might contribute to the minimization of the IS value of the variable of operation. It could be stated that an APS value of the variable of formula that is close to zero causes the formation of the IS value of the variable of operation. In addition, students’ inability to reflect QMT2 achievement levels on formula knowledge levels might have paved the way for the emergence of this IS value. Low QMT2 achievement level might also have caused this. Increasing the QMT2 achievement level can solve the problems created by lowness of it; however it does not necessarily mean that it will be reflected on the knowledge level of the variable of formula. Increasing the knowledge levels of the variables of given-asked and definition may help students link the formula with the subject. A wider solution of the problem pertains to the structure of knowledge. For example, mathematical logic knowledge can be structured as procedural or declarative knowledge, because scientific knowledge is mostly in procedural and declarative knowledge structures.

The finding that there exists a more than 100% difference between students’ achievement levels determined through qualitative and quantitative assessments in the favor of the former is a result of the fact that the knowledge level of this type of assessment had a close value to its achievement level. It might have stemmed from the fact that students looked for the correct answer among five options in the questions of the knowledge test. This opinion is supported by the finding that students’ achievement level had a score close to 1/5=0,20; which is the probability of finding the correct answer. In the qualitative application, on the other hand, the probability is 0,50 since each question is evaluated separately. The reason the possibility of achievement level is 0,39 instead of 0,50 is students’ low knowledge level. The closest knowledge level to the achievement level is that of the variable of operation. Other knowledge levels were almost non-existing; which indicates that their achievement levels depended on the knowledge level of the variable of operation. Students’ tendency to solve proposition questions (mathematical logic) through mathematical operations points to the lack of teaching in the logic dimension of mathematics or lack of knowledge in logical structures of laws of nature. Since students’ knowledge levels depended on the variable of operation in which only mathematical knowledge is used, it is apparent that they know only the mathematical structure of formulae or laws of nature.

This opinion is supported by very low knowledge levels in other variables. Since mathematical logic is a discipline that aims to form the logical structure of laws of nature, important contributions can be made to the elimination of such problems experienced by students by developing teaching methods aimed at teaching this logic and thus teaching the logical aspects of laws of nature. It would be unlikely to take students’ knowledge beyond simply memorizing without teaching them logical structures of subjects. It is a fact that the logical structure of physics is based on mathematical logic. Analyzing a discipline, which has a mathematical logic, through a different logic is a Sisyphean task although it is fun. Such an attempt is the same as measuring time by the metric system. Although the most important job is performed by variables in increasing students’ mathematical logic knowledge and achievement levels, factors that influence achievement are also influential. The factors measured by QMT2 and QMT3 are factors that can be changed.
References


Table 1: Students, scores of variables of QMT1 and achievement levels.

<table>
<thead>
<tr>
<th>Points/Variable</th>
<th>Given-Asked</th>
<th>Definition</th>
<th>Formulas</th>
<th>Operations</th>
<th>Sum Of Variables</th>
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<tbody>
<tr>
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<td>Written</td>
<td>Interview</td>
<td>Difference</td>
<td>Written</td>
<td>Interview</td>
</tr>
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<td>P</td>
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<td>2.00</td>
<td>0.00</td>
<td>10.00</td>
<td>20.00</td>
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<tr>
<td>BGS</td>
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<td>436.00</td>
</tr>
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<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
</tr>
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<td>0.13</td>
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</tr>
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</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.97</td>
<td>0.00</td>
<td>0.97</td>
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</tr>
<tr>
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</tr>
<tr>
<td>QMT3 S</td>
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<td>0.39</td>
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</table>

The response rate to the test 0.15