Efficiency of Some Modified Ratio and Product Estimators Using Known Value Of Some Population Parameters.

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Abstract

Khoshnevisan et al (2007) proposed a general family of estimators for estimating population means using known value of some population parameter(s) which after some substitutions led to some ratio and product estimators initially proposed by Sisodia and Dwivedi (1988), Singh and Tailor (2003), Pandey and Dubey (1988) etc. Adopting Adewara (2006) in improving these estimators observed that all these modified ratio and product estimators perform better than the earlier proposed ones.

Keywords: Ratio, Product, Estimator, Population Parameter, Efficiency.

1. Introduction

Suppose, n pairs (x, y) (i=1,2,…,n) observations are taken on n units sampled from N population units using simple random sampling without replacement scheme, $\bar{X}$ and $\bar{Y}$ are the population means for the auxiliary variable (X) and variable of interest (Y) and $\bar{x}$ and $\bar{y}$ are the sample means based on the sample drawn. Khoshnevisan et al (2007) defined their family of estimators as

$$t = \bar{y}\left[\frac{a\bar{X} + b}{\alpha(a\bar{x} + b) + (1-\alpha)(a\bar{X} + b)}\right]^g$$

where $a(\neq 0)$, b are either real numbers or the functions of the known parameters of the auxiliary variable x such as standard deviation (σ_x), Coefficient of Variation (C_x), Skewness ($\beta_1(x)$), Kurtosis ($\beta_2(x)$) and Correlation Coefficient (ρ).

(i). When α=0, a=0=b, g=0, we have the mean per unit estimator, $t_0 = \bar{y}$ with

$$MSE(t_0) = \left(\frac{N-n}{Nn}\right)\bar{Y}^2 C_y^{\,2} \qquad (1.1)$$

(ii). When α=1, a=1, b=0, g=1, we have the usual ratio estimator, $t_1 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$ with

$$MSE(t_1) = \left(\frac{N-n}{Nn}\right)\bar{Y}^2 (C_y^{\,2} + C_x^{\,2} - 2\rho C_x C_y) \qquad (1.2)$$

(iii). When α=1, a=1, b=0, g=-1, we have the usual product estimator, $t_2 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$ with

$$MSE(t_2) = \left(\frac{N-n}{Nn}\right)\bar{Y}^2 (C_y^{\,2} + C_x^{\,2} + 2\rho C_x C_y) \qquad (1.3)$$

(iv). When α=1, a=1, b=C_x, g=1, we have Sisodia and Dwivedi (1981) ratio estimator, $t_3 = \bar{y}(\frac{\bar{X}+C_x}{\bar{x}+C_x})$ with

$$MSE(t_3) = (\frac{N-n}{Nn})\bar{Y}^2(C_y{}^2 + (\frac{\bar{X}}{\bar{X}+C_x})^2 C_x{}^2 - 2(\frac{\bar{X}}{\bar{X}+C_x})\rho C_x C_y) \quad (1.4)$$

(v). When α=1, a=1, b=C_x, g=-1, we have Pandey and Dubey (1988) product estimator, $t_4 = \bar{y}(\frac{\bar{x}+C_x}{\bar{X}+C_x})$ with

$$MSE(t_4) = (\frac{N-n}{Nn})\bar{Y}^2(C_y{}^2 + (\frac{\bar{X}}{\bar{X}+C_x})^2 C_x{}^2 + 2(\frac{\bar{X}}{\bar{X}+C_x})\rho C_x C_y) \quad (1.5)$$

(vi). When α=1, a=1, b=ρ, g=1, we have Singh, Taylor (2003) ratio estimator, $t_5 = \bar{y}(\frac{\bar{X}+\rho}{\bar{x}+\rho})$ with

$$MSE(t_5) = (\frac{N-n}{Nn})\bar{Y}^2(C_y{}^2 + (\frac{\bar{X}}{\bar{X}+\rho})^2 C_x{}^2 - 2(\frac{\bar{X}}{\bar{X}+\rho})\rho C_x C_y) \quad (1.6)$$

(vii). When α=1, a=1, b=ρ, g=-1, we have Singh, Taylor (2003) product estimator, $t_6 = \bar{y}(\frac{\bar{x}+\rho}{\bar{X}+\rho})$ with

$$MSE(t_6) = (\frac{N-n}{Nn})\bar{Y}^2(C_y{}^2 + (\frac{\bar{X}}{\bar{X}+\rho})^2 C_x{}^2 + 2(\frac{\bar{X}}{\bar{X}+\rho})\rho C_x C_y) \quad (1.7)$$

There are other ratio and product estimators from these families that are not inferred here but this paper will be limited to those ones that made use of Coefficient of Variation (C_x) and Correlation Coefficient (ρ) since the conclusion obtained here can also be inferred on all others that made use of other population parameters such as the standard deviation (σ_x), Skewness ($\beta_1(x)$) and Kurtosis ($\beta_2(x)$) in the same family.

2. On the Modified Ratio and Product Estimators.

Adopting Adewara (2006), $t^*{}_1 = \bar{y}^*(\frac{\bar{X}}{\bar{x}^*})$, $t^*{}_2 = \bar{y}^*(\frac{\bar{x}^*}{\bar{X}})$, $t^*{}_3 = \bar{y}^*(\frac{\bar{X}+C_x}{\bar{x}^*+C_x})$,

$t^*{}_4 = \bar{y}^*(\frac{\bar{x}^*+C_x}{\bar{X}+C_x})$, $t^*{}_5 = \bar{y}^*(\frac{\bar{X}+\rho}{\bar{x}^*+\rho})$ and $t^*{}_6 = \bar{y}^*(\frac{\bar{x}^*+\rho}{\bar{X}+\rho})$, where

where $\bar{x}^*$ and $\bar{y}^*$ are the sample means of the auxiliary variables and variable of interest yet to be drawn with the relationships (i). $\bar{X} = f\bar{x} + (1-f)\bar{x}^*$ and (ii). $\bar{Y} = f\bar{y} + (1-f)\bar{y}^*$. Srivenkataramana and Srinath (1976).

The Mean Square Errors of these Estimators $t^*{}_i$, i = 1,2, …, 6 are as follows:

(i). $MSE(t^*{}_1) = (\frac{n}{N-n})^2 MSE(t_1)$ (2.1)

(ii). $MSE(t^*{}_2) = (\frac{n}{N-n})^2 MSE(t_2)$ (2.2)

(iii). $MSE(t^*{}_3) = (\frac{n}{N-n})^2 MSE(t_3)$ (2.3) (iv). $MSE(t^*_4) = (\frac{n}{N-n})^2 MSE(t_4)$ (2.4)

(v). $MSE(t^*_5) = (\frac{n}{N-n})^2 MSE(t_5)$ (2.5)

and

(vi). $MSE(t^*_6) = (\frac{n}{N-n})^2 MSE(t_6)$ (2.6)

3. Data used.

Since conventionally, for ratio estimators to hold, $\rho > 0$ and also for product estimators to hold, $\rho < 0$. Therefore two data sets are used in this paper, one to determine the efficiency of the modified ratio estimators and the other to determine that of the product estimators as stated below.

Population I: Kadilar and Cingi (2004)

N = 106, n = 20, $\rho = 0.86$, $C_y = 5.22$, $C_x = 2.1$, $\bar{Y} = 2212.59$ and $\bar{X} = 27421.70$

Population II: Maddala (1977)

N = 16, n = 4, $\rho = -0.6823$, $C_y = 0.2278$, $C_x = 0.0986$, $\bar{Y} = 7.6375$ and $\bar{X} = 75.4313$

4. Results

The results obtained from these two data sets are shown in table I below:

Table 1: Showing the estimates obtained for both the proposed and modified ratio and product estimators

Estimator	Population I ($\rho > 0$)	Population II ($\rho < 0$)
t_0	5411349	0.5676
t_1	2542740	-
t_2	-	0.3387
t_3	2542893	-
t_4	-	0.3388
t_5	2542803	-
t_6	-	0.3376
t^*_1	137519.8	-
t^*_2	-	0.03763
t^*_3	137528	-
t^*_4	-	0.03765
t^*_5	137523.1	-
t^*_6	-	0.03751

5. Conclusion

From the above table, we observed that all the modified estimators ($t^*_{i's}$) perform better than the earlier proposed ones ($t_{i's}$) in the family. It has to be stated here also that those ones not discussed in this paper will also behave likewise.

References

Adewara, A.A. (2006): Effects of improving both the auxiliary and variable of interest in ratio and product estimators. Proc. Pakistan Acad. Sci.43(4): 275 – 278.

Kadilar, C. and Cingi, H. (2004). Ratio Estimators in Simple Random Sampling. Appl. Mathe. and Comput. 151, 893-902.

Kadilar, C. and Cingi, H. (2006): Ratio estimators for the population variance in sample and stratified random sampling. Appl. Mathe. and Comput. 173, 1047 – 1059.

Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007): A general family of estimators for estimating population mean using known value of some population parameter(s). Far East Jour. of theor. statist. 22(2), 181 – 191.

Maddala, G.S. (1977): Econometrics. “McGraw Hills Pub. Co.” New York.

Pandey, B.N. and Dubey, Vyas (1988): Modified product estimator using coefficient of variation of auxiliary variate, Assam Statistical Rev., 2(2), 64 – 66.

Singh, H.P. and Tailor, R. (2003): Use of known correlation coefficient in estimating the finite population mean. Statist. in Trans. 6(4). 555 – 560.

Srivenkataramana, T. and Srinath, K.P. (1976): Ratio and Product methods of estimation in sample surveys when the two variables are moderately correlated. Vignana Bharathi 2: 54 – 58.

Sisodia, B.V.S. and Dwivedi, V.K. (1981): A modified ratio estimator using coefficient of variation of auxiliary variable. Journal Ind. Soc. Agril. Statist.,33(2), 13 – 18.