Propagation of Rayleigh Waves through the Surface of an Elastic Solid Medium in the Presence of a Mountain

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Abstract

The paper presents a study of Rayleigh waves propagation due to mountain of arbitrary shape and of width a. The mountain is present in the surface of an elastic solid medium. The method of solution is Wiener-Hopf technique. The reflected, transmitted and scattered waves are obtained by inversion of Fourier transforms. The numerical computations are carried out for the amplitude of the scattered and reflected waves versus wave number and wave length. The scattered waves behave as decaying cylindrical waves at distant points and have a large amplitude near the foot of the mountain.

Keywords: Rayleigh waves, Isotropic medium, reflected and transmitted waves, Wiener-Hopf technique, Fourier transform.

1. Introduction

Rayleigh waves are the surface seismic waves that cause vertical shifting of the earth during an earthquake. These waves are responsible for destruction of the buildings and loss of human lives. Scattering of seismic waves due to irregularities in the surface leads to large amplification and variation in ground motion during earthquakes. Sato [12] has studied the problem of Love wave propagation in case of a surface layer on a solid half space using the Wiener-Hopf technique. Kazi [6] has solved the same problem by an approximate method. Scattering of a compressional wave due to the presence of a rigid barrier in the surface of a liquid half space has been discussed by Deshwal [4]. Momoi [8, 9] has studied the problem of scattering of Rayleigh waves by semicircular and rectangular discontinuities in the surface of a solid half space. The problem of attenuation of Rayleigh waves propagating along an irregular surface of an empty borehole has been investigated by Maximov et al. [7]. Avila-Carrera et al. [2] have considered several crack configurations in order to show the importance of cracks' geometry on Rayleigh wave propagation.

They have used the Indirect Boundary Element Method (IBEM). They [1] have further studied the scattering and diffraction of Rayleigh waves by shallow cracks. In this work, they have used the IBEM to calculate the scattered fields produced by single or multiple cracks near a free surface. Chattopadhyay et al. [3] have studied the reflection of shear waves in visco-elastic medium at parabolic irregularity. They found that amplitude of reflected wave decreases with increasing length of notch and increases with increasing depth of irregularity. In this paper, we aim to study the propagation of Rayleigh wave at the foot of a mountain with its base occupying the region $0 \le x \le a$, $z = 0$ in the surface of a solid half space $z \ge 0$. The shape of mountain is immaterial and it is assumed to be rigid such that there is no displacement across the mountain. The method of solution is the Fourier transformation of the basic equations and determination of unknown functions by the technique of Wiener and Hopf [10].

2. Formulation of the Problem

A mountain of an arbitrary shape and of width a is present in the surface of an elastic solid medium. Its base and the solid half space are given by $0 \le x \le a$ and $z \ge 0$ respectively. The medium is homogeneous, isotropic and slightly dissipative. The problem is two dimensional in zx-plane.

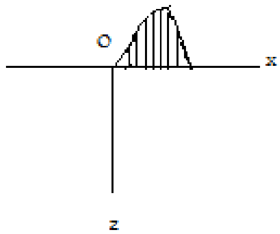

Fig. 1

If the retarding force of the medium is proportional to the velocity then the wave equation is

$$\frac{\partial^2 \bar{\phi}}{\partial x^2} + \frac{\partial^2 \bar{\phi}}{\partial z^2} = \frac{1}{c^2}\left(\frac{\partial^2 \bar{\phi}}{\partial t^2} + \varepsilon \frac{\partial \bar{\phi}}{\partial t}\right) \qquad (2.1)$$

where c is the velocity of propagation and $\varepsilon > 0$ is damping constant. The potential function harmonic in time is

$$\bar{\phi}(x,z,t) = \phi(x,z)e^{-iwt} \qquad (2.2)$$

The equation (2.1) reduces to

$$\left(\nabla^2 + \left(\bar{k}\right)^2\right)\phi(x,z) = 0 \qquad (2.3)$$

where $\bar{k} = \left(\omega^2 + i\varepsilon\omega\right)^{1/2} / c = \overline{k_1} + i\overline{k_2}$

is complex whose imaginary part is small and positive. Let the incident Rayleigh wave be given by

$$\phi_i = B\left(2p_0^{\,2} - k^2\right)\exp\left(ip_0x - \alpha_0 z\right) \qquad (2.4)$$

$$\psi_i = B\left(2ip_0\alpha_0\right)\exp\left(ip_0x - \beta_0 z\right) \qquad (2.5)$$

where p_0 is a root of the Rayleigh frequency equation

$$f(p) = \left(2p^2 - k^2\right) - 4p^2\alpha\beta = 0,\ \alpha = \sqrt{\left(p^2 - \left(\bar{k}\right)^2\right)},\ \beta = \sqrt{\left(p^2 - k^2\right)} \qquad (2.6)$$

$\alpha_0 = \alpha(p_0), \beta_0 = \beta(p_0)$, strikes at the foot of the mountain from the region $x < 0$.

The potential $\psi(x,z)$ satisfies the wave equation

$$\left(\nabla^2 + k^2\right)\psi(x,z) = 0 \qquad (2.7)$$

Let the total potentials be

$$\phi_t(x,z) = \phi(x,z) + \phi_i(x,z) \qquad (2.8)$$

$$\psi_t(x,z) = \psi(x,z) + \psi_i(x,z) \qquad (2.9)$$

The displacement components (u, w) in terms of potential are

$$u = \frac{\partial \phi_t}{\partial x} + \frac{\partial \psi_t}{\partial z},\ w = \frac{\partial \phi_t}{\partial z} - \frac{\partial \psi_t}{\partial x} \qquad (2.10)$$

3. Boundary Conditions

The conditions on the boundaries are

$\phi(x,z),\ \psi(x,z)$ are bounded when $z\to\infty$ (3.1)

u=0 at z=0 for $0\le x\le a$ (3.2)

(iii) w = 0 at z=0 for $0\le x\le a$ (3.3)

(iv) $$2\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial x}\right)-k^2\psi=0,\ z=0,\ x\le 0,\ x\ge a \quad (3.4)$$

(v) $$2\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial z}\right)+k^2\phi=0,\ z=0,\ x\le 0,\ x\ge a \quad (3.5)$$

The conditions (3.2) and (3.3) subject to (2.3) and (2.7) reduce to

$$\phi=-\phi_i=-B(2p_0{}^2-k^2)\exp(ip_0x),\ 0\le x\le a \quad (3.6)$$

$$\psi=-\psi_i=-B(2ip_0\alpha_0)\exp(ip_0x),\ 0\le x\le a \quad (3.7)$$

Let us define the Fourier transform

$$\bar{\phi}(p,z)=\int_{-\infty}^{\infty}\phi(x,z)e^{ipx}dx,\ p=\xi+i\eta$$

$$=\int_{-\infty}^{0}\phi(x,z)e^{ipx}dx+\int_{0}^{a}\phi(x,z)e^{ipx}dx+\int_{a}^{\infty}\phi(x,z)e^{ipx}dx$$

$$=\bar{\phi}_-(p,z)+\bar{\phi}_a(p,z)+\bar{\phi}_{+a}(p,z) \quad (3.8)$$

If for given z, $\phi(x,z)$ and $\psi(x,z)$ have the behavior $\exp(-d|x|)$ as $|x|\to\infty,\ d>0$ then $\bar{\phi}_-(p,z)$ is analytic in $\eta<d$ and $\bar{\phi}(p,z)$ and its derivative w.r.t. z are analytic in the strip $-d<\eta<d$. The transform of $\psi(x,z)$ has the same behavior.

4. Solution of the problem

Fourier transforms of the equations (2.3) and (2.7) give

$$\frac{d^2\bar{\phi}}{dz^2}-\alpha^2\bar{\phi}=0,\ \ \alpha=\pm\sqrt{\left(p^2-\left(\bar{k}\right)^2\right)} \quad (4.1)$$

$$\frac{d^2\bar{\psi}}{dz^2}-\beta^2\bar{\psi}=0,\ \ \beta=\pm\sqrt{\left(p^2-k^2\right)} \quad (4.2)$$

Since $\phi(x,z)$ and $\psi(x,z)$ are bounded as $z\to\infty$ and their transforms are also bounded. The solutions of (4.1) and (4.2) are

$$\bar{\phi}(p,z)=C(p)\exp(-\alpha z) \quad (4.3)$$

$$\bar{\psi}(p,z)=D(p)\exp(-\beta z) \quad (4.4)$$

The signs in radicals for α and β are such that their real parts are positive for all p. Using the notations $\bar{\phi}(p)$, $\bar{\psi}(p)$ for $\bar{\phi}(p,0)$, $\bar{\psi}(p,0)$ etc., from (4.3) we see

$$\frac{\bar{\phi}'(p)}{\alpha}=-\bar{\phi}(p) \quad (4.5)$$

This equation is decomposed

$$\frac{1}{\sqrt{p-\bar{k}}}\left[\frac{\bar{\phi}'_+(p)}{\sqrt{p+\bar{k}}}-\frac{\bar{\phi}'_+(-\bar{k})}{\sqrt{2\bar{k}}}\right]+\bar{\phi}_+(p)+\frac{\bar{\phi}'_-(-\bar{k})}{\sqrt{-2\bar{k}\left(p+\bar{k}\right)}}$$

$$= -\bar{\phi}_-(p) - \frac{1}{\sqrt{p+\bar{k}}}\left[\frac{\bar{\phi}'_-(p)}{\sqrt{p-\bar{k}}} - \frac{\bar{\phi}'_-(-\bar{k})}{\sqrt{-2\bar{k}}}\right] - \frac{\bar{\phi}'_+(\bar{k})}{\sqrt{2\bar{k}(p-\bar{k})}} \qquad (4.6)$$

The L.H.S. of (4.6) is analytic in $\eta > -d$ and the R.H.S. in $\eta < d$. They represent an entire function. Further each side tends to zero as $|p| \to \infty$. By an extension of Liouville's theorem, the entire function is identically zero. Hence, equating each side to zero, we get

$$\bar{\phi}_+(p) = -\bar{\phi}'_+(p)/\alpha + \bar{q}F(p),\ \bar{q} = \bar{\phi}'_+(\bar{k}) = \bar{\phi}'_-(-\bar{k}) \qquad (4.7)$$

$$\bar{\phi}_-(p) = -\bar{\phi}'_-(p)/\alpha - \bar{q}F(p),\ -\bar{q} = \bar{\phi}'_-(\bar{k}) = \bar{\phi}'_+(-\bar{k}) \qquad (4.8)$$

$$F(p) = \frac{1}{\sqrt{2\bar{k}(p-\bar{k})}} - \frac{1}{\sqrt{-2\bar{k}(p+\bar{k})}} \qquad (4.9)$$

In the same way, from (4.4) we get

$$\bar{\psi}_+(p) = -\bar{\psi}'_+(p)/\beta + qG(p),\ q = \bar{\psi}'_+(k) = \bar{\psi}'_-(-k) \qquad (4.10)$$

$$\bar{\psi}_-(p) = -\bar{\psi}'_-(p)/\beta - qG(p),\ -q = \bar{\psi}'_-(k) = \bar{\psi}'_+(-k) \qquad (4.11)$$

$$G(p) = \frac{1}{\sqrt{2k(p-k)}} - \frac{1}{\sqrt{-2k(p+k)}} \qquad (4.12)$$

Fourier transforms of (3.6) and (3.7) give

$$\bar{\phi}_a(p) = -B(2p_0^2 - k^2)\int_0^a e^{i(p+p_0)x}dx = -\frac{B(2p_0^2-k^2)}{i(p+p_0)}\left(e^{i(p+p_0)a} - 1\right) \qquad (4.13)$$

$$\bar{\psi}_a(p) = -B(2ip_0\alpha_0)\int_0^a e^{i(p+p_0)x}dx = -\frac{2p_0\alpha_0 B}{(p+p_0)}\left(e^{i(p+p_0)a} - 1\right) \qquad (4.14)$$

Multiplying (3.4) by e^{ipx} and integrating between $x = -\infty$ and x=0, we get

$$-2ip\int_{-\infty}^{0}\left(\frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial x}\right)e^{ipx}dx - k^2\bar{\psi}_-(p) = -2\left(\frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial x}\right)_0$$

$$-2ip\bar{\phi}'_-(p) + (2p^2 - k^2)\bar{\psi}_-(p) = \left(\frac{\partial\phi_i}{\partial z} - \frac{\partial\psi_i}{\partial x}\right)_0 + 2ip(\psi_i)_0 + 2\alpha_0 B(k^2 - 2pp_0) \qquad (4.15)$$

Similarly (3.5) leads to

$$(2p^2 - k^2)\bar{\phi}_-(p) + 2ip\bar{\psi}'_-(p) = 2ip\left[(p-p_0)(2p_0^2 - k^2) + 2p_0\alpha_0\beta_0\right] \qquad (4.16)$$

Solving (4.8), (4.11), (4.15) and (4.16), we get

$$f(p)\bar{\phi}_-(p) = 2ip\beta\left[q(2p^2-k^2)G(p) - 2ip\alpha\bar{q}F(p)\right]$$

$$+2iB(2p^2-k^2)\left[(p-p_0)(2p_0^2-k^2) + 2p_0\alpha_0\beta_0\right] + 4ip\alpha_0\beta B(k^2 - 2pp_0) \qquad (4.17)$$

$$f(p)\bar{\psi}_-(p) = (2p^2-k^2)\left[-2ip\alpha\bar{q}F(p) + 2\alpha_0 B(k^2 - 2pp_0)\right]$$

$$-2ip\alpha\left[2ip\beta qG(p) + 2iB\left[(p-p_0)(2p_0^2-k^2) + 2p_0\alpha_0\beta_0\right]\right] \qquad (4.18)$$

Integrating (3.4) and (3.5) between x=a and $x = \infty$ after multiplying it by exp(ipx), it is obtained that

$$(2p^2-k^2)\bar{\psi}_{+a}(p) - 2ip\bar{\phi}'_{+a}(p) = -2\alpha_0 B(k^2 - 2pp_0)e^{i(p+p_0)a} \qquad (4.19)$$

$$\left(2p^2-k^2\right)\bar{\phi}_{+a}(p)+2ip\bar{\psi}'_{+a}(p)=-2iB\left[\left(p-p_0\right)\left(2p_0{}^2-k^2\right)+2p_0\alpha_0\beta_0\right]e^{i(p+p_0)a} \quad (4.20)$$

5. Reflected Waves

The factor exp(-ipx) = exp(-iξ x)exp(ηx) in the inverse transforms makes the integrals vanish at infinity in the upper part of the complex plane if $x < 0$ and in the lower part if $x > 0$. For waves in the region $x < 0$, we have

$$\phi(x,z)=\frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}\bar{\phi}_-(p,z)e^{-ipx}dp \quad (5.1)$$

where the line integral (5.1) is in the strip $-d < \eta < d$. The contour is in the upper half of the complex plane where $\bar{\phi}_-(-p,z)$ is analytic and hence

$$\frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}\bar{\phi}_-(-p,z)e^{-ipx}dp=0 \quad (5.2)$$

From (5.1) and (5.2), it is found that

$$\phi(x,z)=\frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}\left[\bar{\phi}_-(p,z)-\bar{\phi}_-(-p,z)\right]e^{-ipx}dp \quad (5.3)$$

To find the integrand, the wave equation (2.3) is integrated from x = ∞ to x=0 after multiplying it by exp(ipx), to find

$$\frac{d^2\bar{\phi}_-}{dz^2}-\alpha^2\bar{\phi}_-=-\left(\frac{\partial\phi}{\partial x}\right)_0+ip(\phi)_0 \quad (5.4)$$

Changing p to –p and subtracting the resulting equation from (5.4), we obtain

$$\left(\frac{d^2}{dz^2}-\alpha^2\right)\left[\bar{\phi}_-(p,z)-\bar{\phi}_-(-p,z)\right]=-2ipB\left(2p_0{}^2-k^2\right)\exp\left(-\alpha_0 z\right) \quad (5.5)$$

The complete solution of (5.5) is

$$\bar{\phi}_-(p,z)-\bar{\phi}_-(-p,z)=L(p)e^{\alpha z}+L_1(p)e^{-\alpha z}+\frac{2ipB\left(2p_0{}^2-k^2\right)e^{-\alpha_0 z}}{p^2-p_0{}^2} \quad (5.6)$$

To find L and L_1, we take z=0 in (5.6) and use (4.17) it is found that

$$\bar{\phi}_-(p,z)-\bar{\phi}_-(-p,z)=2L\sinh\alpha z+\frac{4ip}{f(p)}\Big[\left(2p^2-k^2\right)\beta qG(p)$$
$$+B\left(2p^2-k^2\right)\left(2p_0{}^2-k^2\right)+2B\alpha_0k^2\beta\Big]e^{-\alpha z}$$
$$+2ipB\left(2p_0{}^2-k^2\right)\left(e^{-\alpha_0 z}-e^{-\alpha z}\right)\Big/\left(p^2-p_0{}^2\right) \quad (5.7)$$

Differentiating (5.7) w.r.t. z and putting z = 0, we get

$$\bar{\phi}_-'(p)-\bar{\phi}_-'(-p)=2\alpha L-\frac{4ip\alpha}{f(p)}\Big[\left(2p^2-k^2\right)\beta qG(p)+2B\alpha_0k^2\beta$$
$$+B\left(2p^2-k^2\right)\left(2p_0{}^2-k^2\right)\Big]+2ipB\left(\alpha-\alpha_0\right)\left(2p_0{}^2-k^2\right)\Big/\left(p^2-p_0{}^2\right) \quad (5.8)$$

2L is obtained from here as the left hand member is known from (4.8) and (4.17). We find the integrand in (5.3) to be

$$\bar{\phi}_-(p,z)-\bar{\phi}_-(-p,z)=\frac{2ipB\left(\alpha-\alpha_0\right)\left(2p_0{}^2-k^2\right)}{p^2-p_0{}^2}.\frac{\sinh\alpha z}{\alpha}+\frac{2ipB\left(2p_0{}^2-k^2\right)}{p^2-p_0{}^2}\left(e^{-\alpha_0 z}-e^{-\alpha z}\right)$$
$$+4ip\left[q\left(2p^2-k^2\right)\beta G(p)+B\left(2p^2-k^2\right)\left(2p_0{}^2-k^2\right)+2Bk^2\alpha_0\beta\right]e^{-\alpha z} \quad (5.9)$$

The pole at $p = p_0$ contributes

$$\phi_1(x,z) = \frac{-4p_0\beta_0}{f^{'}(p_0)}\left[q\left(2p_0^{\ 2} - k^2\right)G(p_0) + 2B\alpha_0\left(2p_0^{\ 2} + k^2\right)\right]e^{-ip_0x}e^{-\alpha_0 z} \quad (5.10)$$

This represents the reflected wave in the region $x < 0$. This does not depend upon the width a of the mountain. The reflected shear wave in the region is found to be

$$\psi_1(x,z) = \frac{4p_0\alpha_0}{f^{'}(p_0)}\left[\left(2p_0^{\ 2} - k^2\right)\bar{q}F(p_0) - 4ip_0\alpha_0 B\left(2p_0^{\ 2} - k^2\alpha_0\beta_0\right)\right]e^{-ip_0x}e^{-\beta_0 z} \quad (5.11)$$

6. Scattered and Transmitted Waves

The incident Rayleigh waves are not only reflected but they are scattered also by the surface irregularity. For finding the scattered component, we evaluate the integral in equation (5.3). The integrand in (5.9) has branch points at $p = \bar{k}$ and $p = k$. The branch cuts are given by the conditions $\mathrm{Re}(\alpha) = 0 = \mathrm{Re}(\beta)$. As discussed by Ewing and Press [5], the parts of branch cuts are hyperbolic as shown in figure 2. For contribution along the branch cut we put $p = \bar{k} + iu$, u being small as the main contribution is around the branch point. Along the cut $\mathrm{Re}(\alpha) = 0$ and $\mathrm{Im}(\alpha)$ changes signs along two sides of the cut. Since α is imaginary, α^2 is negative. Therefore

$$\alpha^2 = \left(\bar{k} + iu\right)^2 - \left(\bar{k}\right)^2 = 2iu\left(\bar{k}_1 + i\bar{k}_2\right) - u^2 \quad (6.1)$$

From here, $\alpha = \pm\sqrt{2\bar{k}_2 u} = \pm i\bar{\alpha}, \ \ \bar{k}_1 = 0$ (6.2)

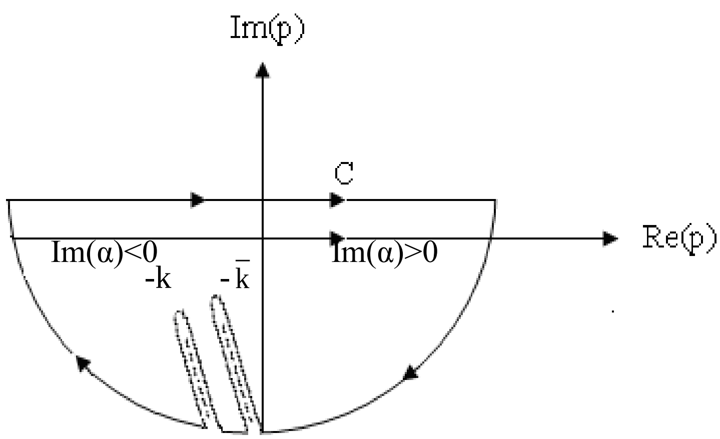

Fig.2 Contour of integration in complex p-plane

Integrating (5.3) along two sides of the branch cut, we find

$$\phi_2(x,z) = \frac{ie^{\bar{k}_2 x}}{2\pi}\int_0^\infty \left[\left[\bar{\phi}_-(p,z) - \bar{\phi}_-(-p,z)\right]_{\alpha = i\bar{\alpha}} - \left[\bar{\phi}_-(p,z) - \bar{\phi}_-(-p,z)\right]_{\alpha = -i\bar{\alpha}}\right]e^{-ux}du \quad (6.3)$$

$$= \frac{2e^{\bar{k}_2 x}}{\pi}\int_0^\infty \left[H_1(u)\sin z\sqrt{2\bar{k}_2 u} + H_2(u)\sqrt{u}\cos z\sqrt{2\bar{k}_2 u}\right]e^{-ux}du \quad (6.4)$$

u is small, $H_1(u)$ and $H_2(u)$ are expanded around u=0 and only $H_1(0)$ and $H_2(0)$ are retained. We have used Laplace integrals as discussed by Oberhettinger and Badii [11].
The scattered waves for the region $x < 0$ are obtained to be

$$\phi_2(x,z) = \frac{\left(2\bar{k}_2\right)^{3/2}}{\sqrt{\pi}}\left[\left(2\left(\bar{k}_2\right)^2 + k^2\right)\frac{z}{x^{3/2}} + 2\left(\bar{k}_2\right)^2\left(\left(\bar{k}_2\right)^2 + k^2\right)\frac{\left(x + \bar{k}_2 z^2\right)}{x^{5/2}}\right].$$

$$\Big[qG(\bar{k})\Big(2(\overline{k_2})^2+k^2\Big)\sqrt{\Big((\overline{k_2})^2+k^2\Big)}-2\alpha_0Bk^2\sqrt{\Big((\overline{k_2})^2+k^2\Big)}$$

$$-iB\left(2p_0^{\,2}-k^2\right)\Big(z(\overline{k_2})^2+k^2\Big)\Big]\exp\left[\overline{k_2}\left(x+\frac{z^2}{2x}\right)\right] \qquad (6.5)$$

when $x \gg z$, then

$$r=\sqrt{x^2+z^2}=x+\frac{z^2}{2x} \qquad (6.6)$$

and the scattered wave is of the form

$$\phi_2(x,z) \sim \exp\left(\overline{k_2}r\right)\Big/\sqrt{r} \qquad (6.7)$$

We now find the waves transmitted to the other side of the mountain. The potential for the region $x>a$ is given by

$$\phi(x,z)=\frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}\bar{\phi}_{+a}(p,z)e^{-ipx}dp \qquad (6.8)$$

$\bar{\phi}_{+a}(-p,z)$ is analytic in the contour which lies in the lower half of the complex plane and so

$$\frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}\bar{\phi}_{+a}(-p,z)e^{2ipa}e^{-ipx}dp=0 \qquad (6.9)$$

From (6.8) and (6.9), we obtain

$$\phi(x,z)=\frac{1}{2\pi}\int_{-\infty+i\eta}^{\infty+i\eta}\left[\bar{\phi}_{+a}(p,z)-e^{2ipa}\bar{\phi}_{+a}(-p,z)\right]e^{-ipx}dp \qquad (6.10)$$

The Fourier transform of the equation (2.3) from x=a to x=∞ gives

$$\frac{d^2\bar{\phi}_{+a}}{dz^2}-\alpha^2\bar{\phi}_{+a}=\left(\frac{\partial\phi}{\partial x}\right)_a e^{ipa}-ip(\phi)_a e^{ipa} \qquad (6.11)$$

To find the integrand in (6.10), we apply the same procedure of finding the integrand in equation (5.3). We find the integrand in (6.10) to be

$$\bar{\phi}_{+a}(p,z)-e^{2ipa}\bar{\phi}_{+a}(-p,z)=\frac{\sinh\alpha z}{\alpha}\Big[\alpha\bar{q}F(p)\left(1-e^{2ipa}\right)$$

$$+iB\left(2p_0^{\,2}-k^2\right)(\alpha-\alpha_0)\left(\frac{1}{p+p_0}+\frac{e^{2ipa}}{p-p_0}\right)\Big]$$

$$-2ipB\left(2p_0^{\,2}-k^2\right)e^{i(p+p_o)a}e^{-\alpha_0 z}\Big/\left(p^2-p_0^{\,2}\right)$$

$$+g(p)e^{-\alpha z}\Big/f(p) \qquad (6.12)$$

where

$$g(p)=2ip\left[-\left(2p^2-k^2\right)q\beta G(p)\left(1+e^{2ipa}\right)-2ip\alpha\beta\bar{q}F(p)\left(e^{2ipa}-1\right)\right]$$

$$+2p\alpha_0\beta B\left(2p_o^{\,2}-k^2\right)\left(\frac{1}{p+p_0}+\frac{e^{2ipa}}{p-p_0}\right)$$

$$+2p_0\alpha_0\beta B\left(2p^2-k^2\right)\left(\frac{1}{p+p_0}-\frac{e^{2ipa}}{p-p_0}\right)$$

$$-2p\alpha\beta B\left(2p_0^{\,2}-k^2\right)\left(\frac{1}{p+p_0}+\frac{e^{2ipa}}{p-p_0}\right)$$

$$-2p_0\alpha_0\beta_0 B\left(2p^2-k^2\right)\left(\frac{1}{p+p_0}-\frac{e^{2ipa}}{p-p_0}\right) \quad (6.13)$$

The pole at $p=-p_0$ in the integrand in (6.10) contributes

$$\phi_3(x,z)=-B\left(2p_0^{\,2}-k^2\right)e^{ip_0x}e^{-\alpha_0 z}-4p_0\left[q\beta_0\left(2p_0^{\,2}-k^2\right)G\left(-p_0\right)\cos 2p_0a\right.$$
$$\left.-2p_0\alpha_0\beta_0 qF\left(-p_0\right)\sin 2p_0 a\right]e^{ip_0(x-a)}e^{-\alpha_0 z}\Big/f'\left(-p_0\right) \quad (6.14)$$

7. Numerical Computations and Discussion of Results

The incident Rayleigh waves are scattered as well as reflected due to the presence of mountain in the surface of an elastic solid medium. The mathematical calculations have been done for Poisson's solids for which $k=\sqrt{3}\bar{k}$ at a point (r=1/2km, z=0) in the region $x<0$ of the free surface. The results are obtained for q=0 and $\alpha=1.8932\bar{k}$. The graphs of amplitude versus the wave number and wave length of the scattered waves has been plotted in figures 3 and 4 respectively. The graphs indicate that the amplitude of the scattered waves depends on the wave number and hence on the wave length of the scattered wave. Also the scattered wave given in equation (6.5) are of the form $\exp\left(-\bar{k}_2 r\right)\big/\sqrt{r}$. These are cylindrical waves which die at distant points from the foot of the mountain. On the free surface (z=0), the scattered waves have the form $\exp\left(-\bar{k}_2 x\right)\big/x^{3/2}$ which is large at the points near the scatterer. Thus the energy of the scattered waves is very large close to the scatterer and diminishes as the wave moves away from it. The transmitted waves in (6.14) depends upon the width a of the mountain. As the distance from the other end of the mountain increases, the transmitted wave decreases exponentially and dies out at distant points. The reflected waves are given by (5.10) and (5.11). The graphs showing the variation of amplitude versus wave number and wave length of reflected Rayleigh waves are shown in figures 5 and 6 respectively. The amplitude of reflected waves increases rapidly as the wave number $\left(\bar{k}\right)$ increases slowly. As the wave length goes on decreasing slowly the amplitude of the scattered wave decreases. With seismic stations spread all over the world, the results of the paper are valid for underground nuclear explosions carried out on either side of the mountains like Himalayas. It helps calculating the amount of energy reflected and scattered at the foot of the mountain and the amount of energy which is transmitted to the other side of the mountain.

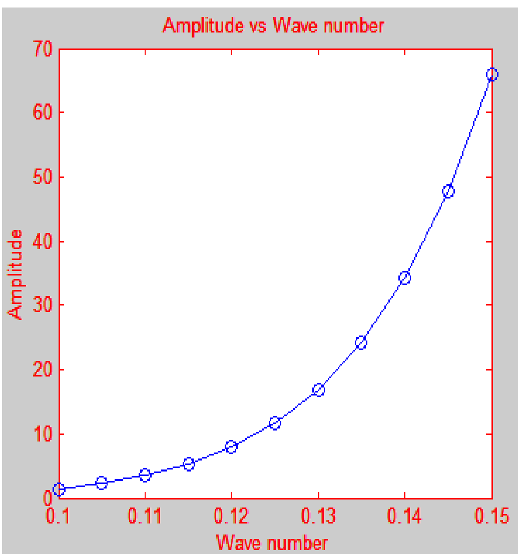

Fig.3 Variation of amplitude vs wave number of scattered waves

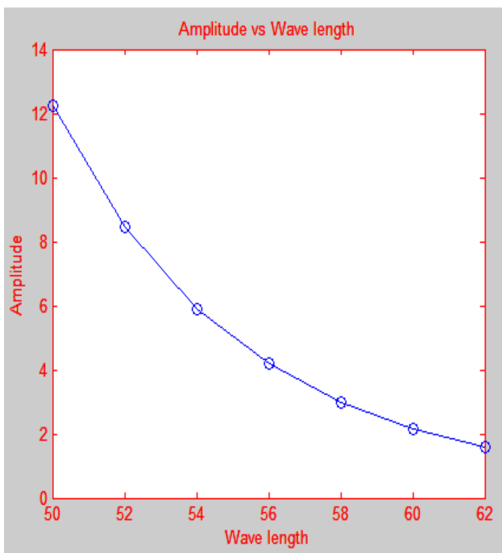

Fig.4 Variation of amplitude vs wave length of scattered waves

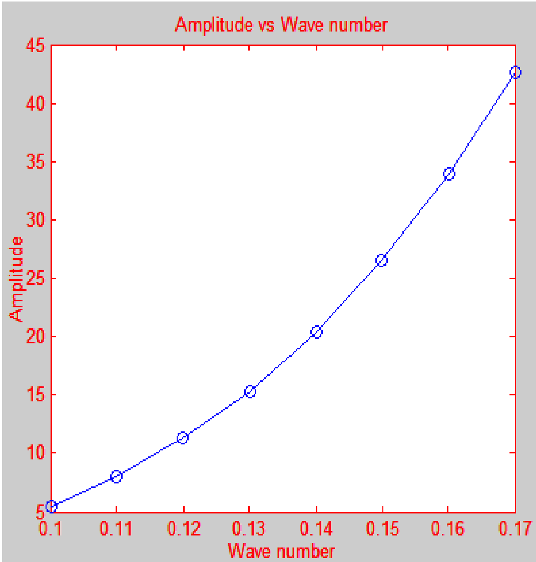

Fig.5 Variation of amplitude vs wave number of reflected waves

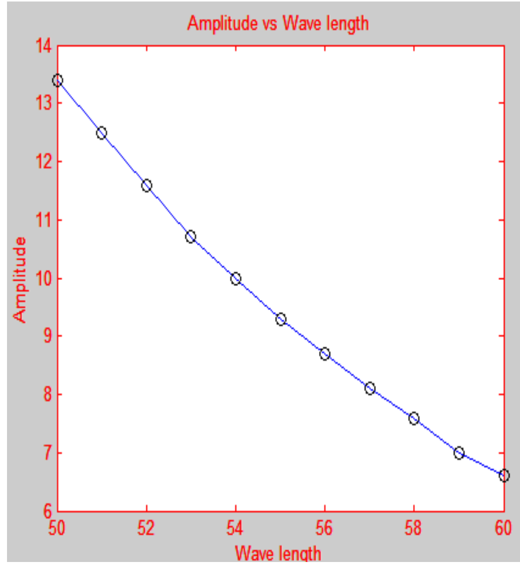

Fig.6 Variation of amplitude vs wave length of reflected waves

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